AIRCRAFT STABILITY AND CONTROL (R15A2110)

COURSE FILE

III B. Tech I Semester

(2018-2019)

Prepared By

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MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

(Autonomous Institution – UGC, Govt. of India)

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VISION

Department of Aeronautical Engineering aims to be indispensable source in Aeronautical Engineering which has a zeal to provide the value driven platform for the students to acquire knowledge and empower themselves to shoulder higher responsibility in building a strong nation.

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PROGRAM EDUCATIONAL OBJECTIVES – Aeronautical Engineering

1. **PEO1 (PROFESSIONALISM & CITIZENSHIP):** To create and sustain a community of learning in which students acquire knowledge and learn to apply it professionally with due consideration for ethical, ecological and economic issues.
2. **PEO2 (TECHNICAL ACCOMPLISHMENTS):** To provide knowledge based services to satisfy the needs of society and the industry by providing hands on experience in various technologies in core field.
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4. **PEO4 (PROFESSIONAL DEVELOPMENT):** To educate the students to disseminate research findings with good soft skills and become a successful entrepreneur.
5. **PEO5 (HUMAN RESOURCE DEVELOPMENT):** To graduate the students in building national capabilities in technology, education and research

PROGRAM SPECIFIC OUTCOMES – Aeronautical Engineering

1. To mould students to become a professional with all necessary skills, personality and sound knowledge in basic and advance technological areas.
2. To promote understanding of concepts and develop ability in design manufacture and maintenance of aircraft, aerospace vehicles and associated equipment and develop application capability of the concepts sciences to engineering design and processes.
3. Understanding the current scenario in the field of aeronautics and acquire ability to apply knowledge of engineering, science and mathematics to design and conduct experiments in the field of Aeronautical Engineering.
4. To develop leadership skills in our students necessary to shape the social, intellectual, business and technical worlds.

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

III Year B. Tech, ANE-I Sem	L	T/P/D	C
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(R15A2110) AIRCRAFT STABILITY AND CONTROL

Objectives:

- To understand the concepts of stability and control of aircraft.
- Develop and understanding of rigid body equations of motion of aerospace vehicle, longitudinal and lateral stability control of aircraft, to know with the aircraft motions and related stability.

UNIT - I:

Aircraft in Equilibrium Flight - Elevator Angle to Trim - Longitudinal Static and Maneuver Stability: Need for controlled flight, Equilibrium, stability, trim, and control- definitions-examples. Longitudinal forces and moments on aircraft in un accelerated flight- contribution of principal components. Equations of equilibrium. Elevator angle required to trim. Longitudinal static stability- definition, Stick fixed neutral point- static margin. Effect of flaps and flight speed on force and moment coefficients, aerodynamic derivatives. Steady, symmetric pull-up maneuvers-equations of motion- pitch rate, pitch damping.

UNIT - II:

Estimation of Aerodynamic Force and Moment Derivatives of Aircraft: Derivatives of axial, normal force components and pitching moment with respect to the flight speed, angle of attack, pitch rate, elevator angle, and flight configuration- effects of flaps, power, compressibility and aero elasticity.

Lateral directional motion- coupling- derivatives of side force, rolling and yawing moments with respect to the sideslip, rate of sideslip, roll rate, yaw rate, aileron, and rudder deflections.

UNIT - III:

Stick Free Longitudinal Stability- Control Forces to Trim, Lateral- Directional Static Stability and Trim: Elevator hinge moments- relation to control stick forces. Hinge moment derivatives, Stick force to trim in symmetric un accelerated flight, maneuvering flight. Stick force gradients- effect of trim speed- role of trim tab. Effect of freeing elevator on tail effectiveness, static and maneuver stability, Elevator- free factor. Stick- free neutral and maneuver points, stability margins- relation with stick force gradients. Aerodynamic and mass balancing of control surfaces. Control tabs- types, function construction.

Lateral- directional static stability, definition, requirements. Equilibrium of forces and moments. Aileron, rudder, elevator and thrust required to trim aircraft in steady sideslip, roll, coordinated turn, engine out condition. Cross wind landings.

UNIT - IV:

Aircraft Equations of motion- Perturbed Motion- Linearized, Decoupled Equations: Description of motion of flight vehicles- systems of reference frames- Euler angles, angles of attack and sideslip- definitions- earth to body axis transformation, Rotation axis system- expressions for linear and angular momenta of rigid body, time derivatives- inertia tensor, components of linear and angular velocities, accelerations. Description of motion as perturbation over prescribed reference flight condition. Equation of motion in perturbation variables. Assumption of small perturbations, first order approximations- linearized equations of motion. Decoupling into longitudinal and lateral-directional motions- conditions for validity- role of symmetry.

UNIT - V:

Longitudinal and Lateral- Directional Dynamic Stability: Linearized longitudinal equations of motion of aircraft- three degree of freedom analysis- characteristic equations- solutions- principal modes of motion- characteristics- time constant, un damped natural frequency and damping ratio- mode shapes- significance.One degree of freedom, two degree of freedom approximations- constant speed (short period), constant angle of attack (long period) approximations- solutions- comparison with three degree of freedom solutions- justification of approximations. Lateral directional equations- three degree of freedom analysis

Text Books:

1. Yechout, T. R. et al., Introduction to Aircraft Flight Mechanics, AIAA education Series, 2003, ISBN 1-56347-577-4.
2. Airplane performance stability and control by Courtland D.Perkins ,Robert E.Hage John wiley& sons

Reference Books:

1. Etkin, B. and Reid, L. D., Dynamics of Flight, 3rd Edition. John Wiley, 1998, ISBN 0-47103418-5.
2. Schmidt, L. V., Introduction to Aircraft Flight Dynamics, AIAA Education Series, 1998, ISBN A-56347-226-0.

3. McCormick, B. W., Aerodynamics, Aeronautics and Flight Mechanics, 2nd Edition., Wiley India, 1995, ISBN 978-]
4. Nelson, R. C., Flight Stability and Automatic Control, 2nd Edition., Tata Mc Graw Hill, 2007, ISBN 0-07-066110-3.

Outcomes:

- An understanding of the static stability of aircraft.
- An understanding of dynamic response of aircraft.
- To assess the requirement of control force and power plant.

III B.TECH I SEMESTER – AERONAUTICAL ENGINEERING
AIRCRAFT STABILITY AND CONTROL (R15)
MODEL PAPER – I
Total marks: 75

PART A **Marks: 25**

i. All questions in this section are compulsory
ii. Answer the question in brief.

1) Explain the termsStatic and Maneuver Stability? [3M]
2) What is the equilibrium condition of an aircraft? [2M]
3) What is stability derivatives? [3M]
4) Explain angle of yaw and angle of side slip? [2M]
5) Define Stick fixed neutral point? [3M]
6) Explain about one engine inoperative condition [3M]
7) Define pitch rate, and pitch damping? [2M]
8) Write down the expressions for stability derivatives of an airplane [3M]

in pitch, yaw and roll.
9) Define damping ratio and natural frequency [2M]
10) What are the forces acting on a flight with respective to stability and control? [2M]

PART B Marks: 5x10= 50

Answer only one question among the two questions in choice.
i. Each question answer (irrespective of the bits) carries 10M.

1) Explain the significance of Routh's discriminant.

Or

2) Drive an expression for the tail contribution to the pitching moment of an aircraft (assume it1 is the wing setting angle and it2 is the tail setting angle).
3) Derive an expression for aircraft side force with respect to the side slip, rate of side slip , roll rate , yaw rate, aileron , rudder deflections

Or

4) Define stability derivatives with the representation of aerodynamic forces and moments
5) Derive the equation for elevator free factor with required sketches

Or

6) Explain the use of hinge moments in determining stick force to be applied by the pilot in unaccelerated flight of the airplane .Show that with $dFs/dv<0$,the airplane statically unstable

7) Derive the equations of motion of a rigid body subjected to inertial forces and moments illustrate with sketches

Or

8) With a neat sketch show the axes system associated with an airplane

9) Describe the motion of airplane after it has entered in to spinning. What are the causes of airplane getting in to spin? How does the pilot make recovery from spin

Or

10) Define the term longitudinal dynamic stability of the airplane. Explain if an airplane when possessing static longitudinal stability will as well be dynamically stable. Make use of the stability quartic equation and sketches accompanied by plots illustrating typical modes of motion in support of your answer

III B.TECH I SEMESTER – AERONAUTICAL ENGINEERING
AIRCRAFT STABILITY AND CONTROL (R15)
MODEL PAPER – 2

Total marks: 75
PART A Max Marks: 25

1. Define static stability and dynamic stability. [2M]
2. What do you mean by degree of freedom? [3M]
3. What is meant by aileron reversal? [2M]
4. What is snaking. Sketch the snaking motion of an aircraft. [2M]
5. Difference ranging from stick fixed and stick free. [3M]
6. What is meant by `Weather Cock Stability'? [2M]
7. What are the condition for longitudinal static stability? [2M]
8. How is dihedral useful for lateral stability. [3M]
9. What is meant by roll mode? [3M]
10. What is meant by phugoid oscillation? Discuss. [3M]

PART B Marks: 5x10= 50

Answer only one question among the two questions in choice.

i. Each question answer (irrespective of the bits) carries 10M.

1) Explain the significance of Routh's discriminant.

Or

2) Discuss in detail the power effects on longitudinal static stability.

3) Write short notes on. Variable incidence tail plane and Adverse Yaw

Or

4) What is the coupling ranging from rolling and yawing moments, discuss with suitable examples.

5) Explain stick force per g in detail

Or

6) Derive the neutral point equation for stick free condition with respective fig

7) Explain the position and orientation of an aircraft relative to earth and describe it in terms of Euler's angles

Or

8) Derive the equation of aircraft force equations and moment equations

9) Explain the following phenomenon (a) Dutch roll (b) Spiral instability (c) spin

Or

10) Write a short notes on (a) stability derivatives in longitudinal dynamic stability and

(b) Stability quartic

III B.TECH I SEMESTER – AERONAUTICAL ENGINEERING
AIRCRAFT STABILITY AND CONTROL (R15)
MODEL PAPER – 3

Total marks: 75

PART A Marks: 25

1) What is the criterion for static longitudinal stability [3M]
2) Graphically represent a system which is statically stable but dynamically unstable [3M]
3) Define aileron reversal [2M]
4) Explain about lateral control [2M]
5) Define cross wind landings [3M]
6) Plot the variation of hinge moment with control deflections & hinge moment

with angle of attack [3M]
7) Define stability axis system [2M]
8) Define Euler angle rates and body axis rates [3M]
9) What is meant by Dutch roll mode [2M]
10) Explain short period oscillation [3M]

PART B Marks: 5x10= 50

i. Answer only one question among the two questions in choice.
ii. Each question answer (irrespective of the bits) carries 10M.

1) Explain in detail about the elevator control power with sketches .Derive the equation (dcm/dcl)tail = -atvηt

Or
2) Derive the equations for maneuverability – the elevator angle and control force required to hold the airplane in a steady pull-up with load factor with required sketches
3) Describe the derivatives of yawing moment of an aircraft with respect to the side slip, rate of side slip, roll rate, yaw rate , aileron , rudder deflections

Or
4) Explain the difference between aerodynamic coefficients and aerodynamic derivatives .Give four pairs of examples with explanation
5) Explain stick force gradients in detail

Or
6) Derive the equation for the control effectiveness of elevator

7) Derive the longitudinal linearized equations of motion with small perturbation approach

Or

8) With the first–order approximation of applied aero forces and moments get the equations for longitudinal and lateral – directional perturbed forces and moments

9) Describe Dutch roll and spiral instability

Or

10) Explain how stability quartic helps in studying the dynamic stability of an aircraft

III B.TECH I SEMESTER – AERONAUTICAL ENGINEERINGAIRCRAFT STABILITY AND CONTROL (R15)
MODEL PAPER – 4
Total marks: 75

PART A Marks: 25

1) Write a short notes on control of aircraft [2M]
2) Define neutral point, Balance or Equilibrium [3M]
3) How is dihedral useful for lateral stability [2M]
4) Define yawing moment [2M]
5) Write down the equation for elevator floating angle [3M]
6) Plot the variation of hinge moment with control deflections & hinge moment

with angle of attack [3M]
7) Explain lateral –directional applied forces and moments [3M]
8) Explain the four step approach summarizes the linearization technique [3M]
9) Write a short notes on two degree of freedom Dutch roll approximations [2M]
10) Write a short notes on two-degree of freedom spiral approximation [2M]

PART B Marks: 5x10= 50

i. Answer only one question among the two questions in choice.
ii. Each question answer (irrespective of the bits) carries 10M.
11) Derive the equation for contribution of wing of a aircraft pitching moment with neat sketches

Or
12) Define static margin , elevator power , and what is control effectiveness factor
13) Explain about the representation of aerodynamic forces and moments

Or
14) Derive the equation for aircraft side force with required figs
15) Define hinge moments of aerodynamic surfaces. Derive an expression for the floating angle of an elevator

Or
16) Define the terms ‘floating tendency and restoring tendency’. What is floating of a control surface? Describe ways and means to alleviate or control these hinge moments by an arrangement known as set – back hinge line
17) Derive a six step procedure that used to build up the response side of the three moment equations

Or

18) Summarize the small perturbation approach and develop the linearized aircraft equations of motion

19) Explain about spiral mode and roll subsidence

Or

20) Discuss the dynamic stability aspects of an aircraft considering its linearized longitudinal equations of motion being analyzed under three- degrees, two-degrees and one degrees of freedom assumptions

III B.TECH I SEMESTER – AERONAUTICAL ENGINEERING
AIRCRAFT STABILITY AND CONTROL (R15)
MODEL PAPER – 5
Total marks: 75

PART A Marks: 25

1) What is the criterion for static longitudinal stability [3M]
2) Graphically represent a system which is statically stable but dynamically unstable [3M]
3) Define aileron reversal [2M]
4) Define angle of yaw and angle of side slip [2M]
5) Define hinge moment and write down the expression for hinge moment

coefficient [3M]
6) Explain about one engine inoperative condition [3M]
7) Define body axis system [2M]
8) Explain about the coordinate transformation [3M]
9) What is meant by phugoid oscillation [3M]
10) What is meant by roll mode [2M]

PART B Marks: 5x10= 50

i. Answer only one question among the two questions in choice.
ii. Each question answer (irrespective of the bits) carries 10M.

1) Explain the contribution of fuselage of aircraft pitching moment
Or
2) Derive the axis component of entire airplane and explain about the basic longitudinal
Forces with required sketches

3) Explain about the estimation of aerodynamic force and moment derivatives of aircraft
Or
4) Explain about the lateral directional static stability and aero elasticity

5) Explain stick force per g in detail
Or
6) Derivethe neutral point equation for stick free condition with respective fig

7) Explain the position and orientation of an aircraft relative to earth and describe it in terms of
Euler's angles

Or

8) Derive the equation of aircraft force equations and moment equations

9) Explain the following phenomenon (a) Dutch roll (b) Spiral instability (c) spin

Or

10) Write a short notes on (a) stability derivatives in longitudinal dynamic stability and

(b) Stability quartic

Code No: R15A2110

R15

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY
(Autonomous Institution – UGC, Govt. of India)
III B.Tech I Semester Regular Examinations, November 2017
Aircraft Stability and Control
(AE)

Roll No										

Time: 3 hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carriers 25 marks and Answer all questions.
Part B Consists of 5 SECTIONS (One SECTION for each UNIT).
Answer FIVE Questions, Choosing ONE Question from each SECTION and each Question carries 10 marks.

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PART – A **(25 Marks)**

(a) How would increasing the tail volume, ratio charge the longitudinal static stability of a conventional aircraft? 2M

(b) Explain the trade-off between stability and maneuverability? 3M

(c). What is meant by pure pitching motion? 2M

(d) Write about the three types of controls with reference to C.G of an airplane? 3M

(e) What are the two necessary criteria for longitudinal balance and static stability? 2M

(f) Explain about lateral control? 3M

(g) Describe about system reference fames. 2M

(h) Define pitch rate and pitch damping. 3M

(i) Define mode shapes and undamped frequency. 2M

(j) What is snaking and sketch the snaking motion of an aircraft ?
3M

PART – B **(50 Marks)**

SECTION – I

2. Discuss about the following with respect to stability and control
(a) Balance or equilibrium (b) stability (c) Static margin and elevator power 4+2+4= 10M

(OR)

3. Derive the Euler's equations of motion of complete aircraft.
10M

SECTION – II

4. Describe the changes that takes place in the forces and moment when the angle of attack of the airplane is increased.
10M

(OR)

5. Describe about the
(a) Stability derivatives describing the side force due to yaw rate.
5M
(b) Stability derivatives with respect to roll rate 'p'.
5M

SECTION – III

6. Derive the expression for the stick fixed neutral point and stick free Neutral point?

10M

(OR)

7. Derive the expression for the pedal force of the rudder of an airplane as a function of hinge moment coefficients?
10M

SECTION – IV

8. Explain in detail the two distinct types of longitudinal modes, required in describing the motion of aircraft. When the aircraft is not is performed about the roll and yawing axis?
10M

(OR)

9. Explain about first order approximation of applied aero forces and moments of an aircraft and discuss its usefulness in explaining the behavior of an aircraft for a disturbance. 10M

SECTION – V

10. Explain the following phenomenon
a)Dutch roll b) spiral instability c) spin d) Euler's angles (4+2+2+2)M

(OR)

11. Discuss the dynamics stability aspects of an aircraft, considering its linearized longitudinal equations of motion being analyzed under three degree and two degree of freedom assumptions.
10M

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Unit 1

Introduction

Keywords : Importance of stability and control analysis ; brief historical background ; basic concepts – static stability, dynamic stability, longitudinal, lateral and directional stability, control fixed and control free stability ; controllability; subdivisions of the subject; course outline.

Topics

1.1 Opening remarks

1.2 Brief outline of historical developments

1.2.1 Early developments

1.2.2 Subsequent developments

1.3 Basic concepts about airplane stability and control

1.3.1 Stable, Unstable and neutrally stable states of equilibrium

1.3.2 Types of motions following of disturbance – subsidence, divergence, neutral stability, damped oscillations, divergent oscillation and un-damped oscillation.

1.3.3 Static stability and dynamic stability

1.3.4 Recapitulation of some terms – body axes system, earth fixed axes systems, attitude, angle of attack and angle of sideslip

1.3.5 Longitudinal and lateral stability

1.3.6 Control fixed and control free stability

1.3.7 Subdivisions of stability analysis

1.4 Controllability

1.5 General remarks

1.5.1 Examples of stability in day-to-day life

1.5.2 Airplane stability depends on flight condition

1.5.3 Stability and controllability are not the same

1.5.4 Stability is desirable but not necessary for piloted airplanes

1.5.5 Small disturbance analysis of stability

1.5.6 Rigorous definition of terms

1.6 Course content

1.7 Back ground expected

References

Exercises

1.1 Opening remarks

In the introduction to flight dynamics-I, it was mentioned that flight dynamics deals with the motion of objects moving in earth's atmosphere. The attention in that course and the present one is focused on the motion of the airplane. Helicopters, rockets and missiles are not covered.
Flight dynamics is subdivided into two main topics viz.
(a) airplane performance and
(b) airplane stability and control.
Airplane performance was dealt with in flight dynamics-I (Aircraft Performance). This course, deals with stability and control.
Stability and control of airplane is one of the fascinating subjects in aeronautics. This is because of the following reasons.

- A detailed theoretical analysis of the stability and control of an airplane requires sophisticated mathematical techniques while its experimental assessment calls for sophisticated wind tunnel and flight test techniques.
- Hence, this topic has an appeal for both the theoretician and the experimentalist.
- Further, the importance of stability and control analysis can be judged from the fact that the lack of adequate stability and control was the cause for the failure of early heavier than air machines to sustain themselves in air.

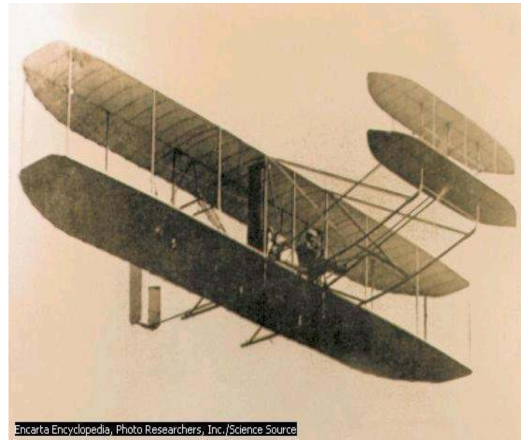
The historical developments in this subject are briefly dealt with in the next section, which is followed by
(a) discussion of the basic concepts of airplane stability and control,
(b) course content and
(c) back ground expected from the reader.

1.2 Brief outline of historical developments

1.2.1 Early developments

The first attempts to study the stability of vehicles in flight were made by Sir George Cayley (1774-1857) who also carried out experiments on models of gliders with horizontal tail and rudder.
By the 1880's, I.C. engines were available which were lighter than the earlier engines. However, inadequate understanding of stability and control delayed the first successful flight of a powered vehicle. Otto Lilienthal (1848-1896) during 1890-1895 and Wilbur Wright (1867- 1912) and Orville Wright (1871-1948) during 1900-1903 carried out a number of experiments on hang gliders and gliders, which gave a better understanding of the stability and control. This led to the

first successful flight on Dec.17, 1903. The name of this airplane was Wright flyer (Fig.1.1). It had a canard surface ahead of the wings for control of the pitching motion, vertical rudder for directional control while control in roll was obtained by warping the wings.



(From : http://www.old-picture.com/ (From :http://fractal-vortex.narod.ru/
wright-brothers-index-001.htm)
productive_forces/past_images/wright2.jpg)

Fig.1.1 Two views of Wright Flyer

The first airplane with ailerons (Fig.1.2) was built in 1907 by Louis Blériot (1872-1936). It was also a monoplane. The first airplane with horizontal tail at the rear (Fig.1.3) was constructed in 1909 by A. Verdon-Roe (1877-1970).

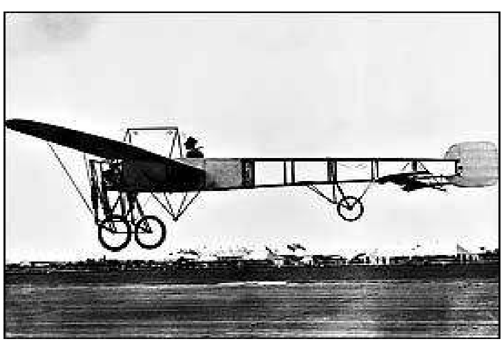


Fig.1.2 Two views of Louis Blériot‟s airplane

Fig.1.3 Airplane of A. Verdon-Roe

As regards the theoretical analysis, F.W. Lanchester (1868-1946) gave ideas about stability in his book entitled **"Aerodonetics"** published by Archibald Constable in 1908. He also mentioned about motion following longitudinal disturbance and called it phugoid.

In 1911, G H Bryan published a book entitled „Stability in aviation", published by Macmillan in which he presented the mathematical analysis of the flight following a disturbance. It may be added that in the equilibrium state the resultant forces and moments acting on the airplane are zero. Any event altering this state is a disturbance. It could be for example, (a) movement of airplane controls by the pilot or (b) inputs beyond pilot"s control like gust of air.

The equations derived by Bryan still form the basis of stability analysis.

1.2.2 Subsequent developments

In the 1930's, the flying qualities of the airplane were studied. These (flying qualities) are based on the opinion of the pilots regarding the amenability of the airplane to perform chosen tasks with precision and without undue effort on the part of the pilot. These were correlated to features of the motion like frequency of oscillation, damping etc. and finally to the geometric features of the airplane like area of horizontal tail, area of vertical tail and dihedral.

In the 1940's automatic control of airplanes became possible. An airplane with automatic control has sensors to detect the linear and angular accelerations and changes in flight path. Once the changes have been detected, the control surfaces are deflected automatically depending on the quantity sensed and the corrections needed. An airplane with automatic control is equivalent to an airplane with a different level of stability. By changing the ratio of input to the output of the automatic control system, it was possible in 1950's to have airplanes with variable stability.

Supersonic flight became possible in 1950's after gaining an understanding of the changes in drag coefficient, lift coefficient and pitching moment coefficient when flight Mach number (M) changes from subsonic to supersonic. These changes also affect the stability of the airplane. It was also understood that the adverse effects of these changes can be alleviated by use of wing sweep (Fig.1.4).

In 1980's airplanes with fly-by-wire technology were available. In this technique the movement of the control stick or pedals by the pilot is transmitted to a digital computer. The input to the computer is processed along with the characteristics of the airplane and the actuators of the controls are operated so as to give optimum performance.

(a)Supersonic transport- Concorde

(b) Fighter-MIG- 29M

Fig.1.4 Supersonic airplanes

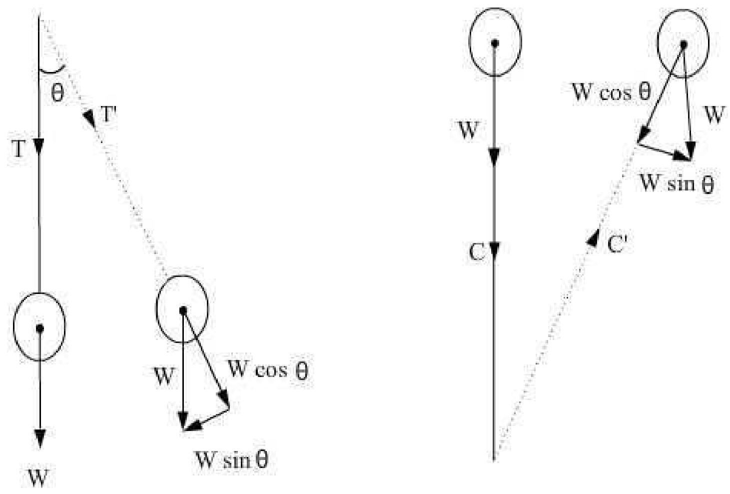
Recent developments include ***relaxed static stability and control configured vehicle (CCV). Relaxed static stability is used in fighter airplanes to improve their performance***. The light combat aircraft (LCA) designed and developed in India has this feature. In a control configured vehicle, the control surfaces and flaps are automatically deployed when the airplane changes from one flight to another. With CCV the structural weight, size of the wing and size of control surfaces can be reduced to an optimum level while achieving greater maneuverability of the airplane.
For further details see Refs. 1.1 and 1.2.

1.3 Basic concepts about airplane stability and control

While carrying out the performance analysis in flight mechanics-I, various equilibrium states were considered. For example, in a steady level flight, an airplane is considered to be flying at a constant altitude along a straight line at constant speed. The equilibrium equations for this flight give the lift and the thrust required during the flight. Subsequent analysis of these equations gives important items of performance. It may be pointed out that these analyses tacitly assume that the airplane will continue to fly in the equilibrium state. However, in actual practice it is noticed that among the various equilibrium states that we can imagine, some are not observable. To illustrate this, consider the following example.
One can imagine a chalk piece to rest in equilibrium on its narrow rounded end on a smooth horizontal table. However, no one has seen this equilibrium. The reason for this is that while imagining the equilibrium it is tacitly assumed that the chalk piece is rotationally symmetric about the center point of the rounded end and that there are no disturbances (e.g. small current of air). On the other hand, the chalk piece can be made to stand on the table on its flat, broad end. It will remain standing even in the presence of a small current of air like a gentle blowing.
Of course, blowing hard at the chalk piece will topple it. This brings us to the following important observations.
(a) ***There are equilibrium states from which, when a system is disturbed slightly over a short period, it will return to the equilibrium state. In other case it will not. The former are termed stable states of equilibrium and the later as unstable states of equilibrium.***
(b) When the disturbance is large, the system may not come back to the equilibrium state. Further, analysis of the case of large disturbance is more complicated.(section 7.7 may be referred to for further details)

1.3.1 Stable, unstable and neutrally stable states of equilibrium

To explain the concepts of stable and unstable equilibria, let us consider the example of a pendulum.



(a) Bob at the bottom – state „A‟ (b) Bob at the top – state „B‟

Fig.1.5 Equilibrium states and stability of a pendulum

Figure 1.5a shows the pendulum in a state referred to as 'A'. In this state, the weight (W) of the bob is supported by the tension (T) in the rod. Let the pendulum be disturbed, so that it makes an angle θ to the original position. In this disturbed position, the weight of the bob has components $Wcos\theta$ and $Wsin\theta$. The component $Wcos\theta$ is balanced by the tension (T) in the rod whereas the unbalanced component $Wsin\theta$ causes the pendulum to move towards the undisturbed position. While returning to the equilibrium position, the bob may overshoot that position.
However, when there is friction at the hinge and/or damping due to the medium in which the pendulum moves, it (pendulum) will eventually come back to its original equilibrium position. Thus, the ***equilibrium 'A' is a case of stable equilibrium***.
In equilibrium state 'B' as shown in Fig.1.5 (b), the weight of the bob is balanced by compression (C) in the rod. Let the pendulum be disturbed, so that it makes an angle θ to the original position. In this disturbed position, the weight of the bob has components $Wcos\theta$ and $Wsin\theta$. The component $Wsin\theta$ in this case tends to move the pendulum away from its equilibrium position. Hence, ***equilibrium 'B' is unstable***.
Apart from the stable and unstable equilibria, there is *a third state called neutrally stable* equilibrium. It is defined as follows.
If a system, when disturbed slightly from its equilibrium state, stays in the disturbed position (neither returns to the equilibrium position nor continues to move away from it), then, it is said to be in neutrally stable equilibrium. In the above example of the pendulum, if the static friction at the hinge is very large, then, on being disturbed from the equilibrium position, it will remain in the disturbed position.

1.3.2 Possible types of motions following a disturbance – subsidence, divergence, neutral stability, damped oscillation, divergent oscillation and undamped oscillation

After a system has been disturbed from it's equilibrium position, it's subsequent motion will be like any one on the six types shown in Fig.1.6. For the sake of the subsequent discussion, it is assumed that initially the disturbance is positive.

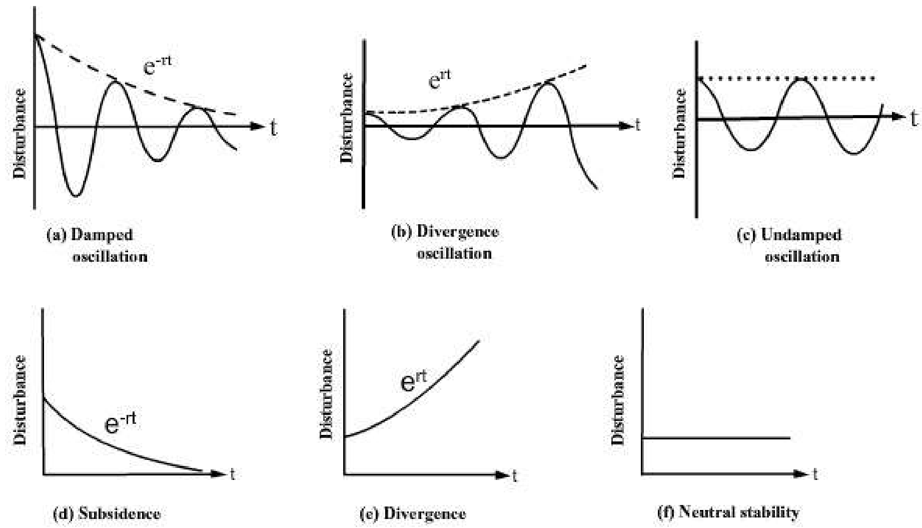


Fig.1.6 Types of motion following a disturbance

i) **Figure 1.6a** shows a damped oscillation. In this case the system while returning to the equilibrium position goes beyond the undisturbed state towards the negative side. However, the amplitude on the negative side is smaller than the original disturbance and it (amplitude) decreases continually with every oscillation. Finally, the system returns to the equilibrium position. The time taken to return to the equilibrium position depends on the damping in the system. An example of this is the motion of pendulum (Fig.1.5 a) when there is friction at the hinge or the pendulum moves in a fluid (air or water). The friction at the hinge or that between the bob and the fluid results in damping.

ii) **Figure 1.6b** shows the divergent oscillation. In this case also the system shows an oscillatory response but the amplitude of the oscillation increases with each oscillation and the system never returns to the equilibrium position. It may even lead to disintegration of the system. An example of this is the divergent oscillation of telephone cables. During winter, in cold regions, ice forms on the telephone cables. Sometimes the cross section of the cable with ice becomes un-symmetric. Such a cable when it starts oscillating may sometimes get into divergent oscillation leading to snapping of cables. Divergent oscillations are seldom encountered. The practical systems are designed such that they do not get into divergent oscillations.

iii) ***Figure 1.6c*** *shows the un-damped oscillation. In this case also the system shows an oscillatory response but the amplitude of the oscillation remains unchanged and the system never returns to the equilibrium position. An example of this situation is the ideal case of the pendulum motion (Fig.1.5a), when the hinge is frictionless and the pendulum oscillates in vacuum.*

iv) When a system returns to its equilibrium position without performing an oscillation, the motion is said to be a subsidence (Fig.1.6d). An example of this could be the motion of a door with a hydraulic damper. In the equilibrium position the door is closed. When someone enters, the equilibrium of the door is disturbed. When left to itself the door returns to the equilibrium position without performing an oscillatory motion.

v) *Conversely, when the system continuously moves away from the equilibrium position, the motion is called divergence* (Fig.1.6e).
vi) If the system stays in the disturbed position (Fig.1.6f), then the system is said to have neutral stability.

1.3.3 Static stability and dynamic stability

In the cases illustrated by Fig.1.6 a, b, c and d, it is observed that, as soon as the system is disturbed, it tends to return to the undisturbed position. Such systems are called statically stable.
For the case in Fig.1.6e, the tendency of the system, immediately after the disturbance, is to turn away from the equilibrium position. Such a system is said to be ***statically unstable****.*
When the tendency of the system, after the disturbance, is to stay in the disturbed position, then it is said to have **neutral static stability**.
Even when the system has a tendency to go towards the undisturbed position (cases 1.6a, b, c and d), it may not return to the equilibrium position as in the cases shown in Fig.1.6 b & c namely divergent oscillation and undamped oscillation. Only when the system finally returns to the equilibrium position, the system is said to be ***dynamically stable****.*
Otherwise, it is dynamically unstable. With this criterion, the damped oscillation and subsidence are the only dynamically stable cases.
Remarks:

i) The definitions of the terms static stability and dynamic stability are as follows:
ii) **Static Stability:** A system is said to be statically stable when a small disturbance causes forces and moments that tend to move the system towards its undisturbed position. If the forces and moments tend to move the system away from the equilibrium position, then the system is said to be statically unstable. In the case of a system having neutral static stability, no forces or moments are created as a result of the disturbance.
iii) **Dynamic Stability:** A system is said to be dynamically stable if it eventually returns to the original equilibrium position after being disturbed by a small disturbance.
iv) It is obvious from the above discussion that for a system to be dynamically stable, it must be statically stable. Table 1.1 categories the cases in Fig.1.6 as
regards the static stability and dynamic stability.

Case	re	c stability	amic stability
ped oscillation	1.6a	Yes	Yes
rgent oscillation	1.6b	Yes	No
amped oscillation	1.6c	Yes	No
idence	1.6d	Yes	Yes
rgence	1.6e	No	No
ral stability	1.6f	No	No

Table 1.1 Static and dynamic stability

v) The distinction between static stability and dynamic stability is of special significance in aeronautical applications as the analysis of static stability is much simpler than that of the dynamic stability. This can be explained as follows.
The disturbance to an airplane in flight due to a gust may change its angle of attack (α) or sideslip (β) or bank (φ) or the thrust output. Now, these changes may produce changes in

aerodynamic forces and moments. If these forces and moments tend to bring the airplane to the original state, then the airplane is statically stable.

Thus, to assess the static stability, one needs only to examine the aerodynamic / propulsive forces and moments brought about at the time the disturbance is applied. On the other hand, to examine the dynamic stability of the airplane, one has to consider the subsequent motion which involves accelerations and hence, the inertia forces.

Further, the dynamic stability analysis requires solution of the equations of motion taking into account the changes, with time, in aerodynamic forces and moments due to changes in α, β, φ, and the linear and angular velocities etc. of the airplane. These quantities denoting changes in aerodynamic forces and moments due to aforesaid changes are called aerodynamic / stability derivatives. Hence, in aeronautical engineering practice first the static stability is ensured by providing adequate areas of horizontal tail and vertical tail and the dihedral angle. Subsequently, the dynamic stability analysis is carried out to ensure that there is adequate damping.

1.3.4 Body axes system, Earth fixed axes system, attitude, Angle of attack and Angle of sideslip

At this stage a brief discussion on body axes system, attitude, angle of attack and angle of sideslip would be helpful

I) Body axes system

To formulate and solve a problem in dynamics we need a system of axes. To define such a system, we note that an airplane is nearly symmetric in geometry and mass distribution about a plane which is called the plane of symmetry (Fig.1.7). This plane is used for defining the body axes system.

Figure 1.8 shows a system of axes $(OX_bY_bZ_b)$ fixed on the airplane which moves with the airplane and hence called body axes system. The origin 'O' of the body axes system is the center of gravity (c.g.) of the body which, by assumption of symmetry, lies in the plane of symmetry. The axis OX_b is taken as positive in the forward direction. The axis OZ_b is perpendicular to OX_b in the plane of symmetry, positive downwards. The axis OY_b is perpendicular to the plane of symmetry such that $OX_bY_bZ_b$ is a right handed system.

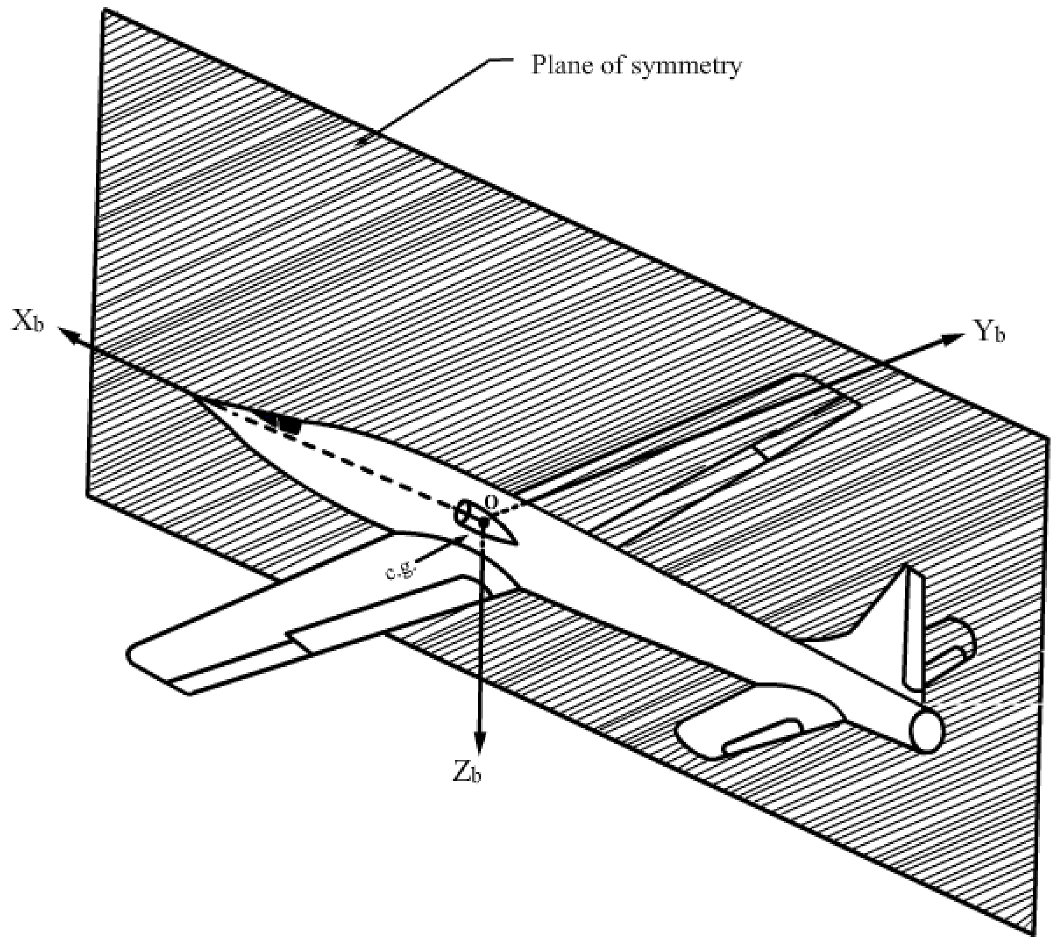


Fig.1.7 Plane of symmetry and body axis system

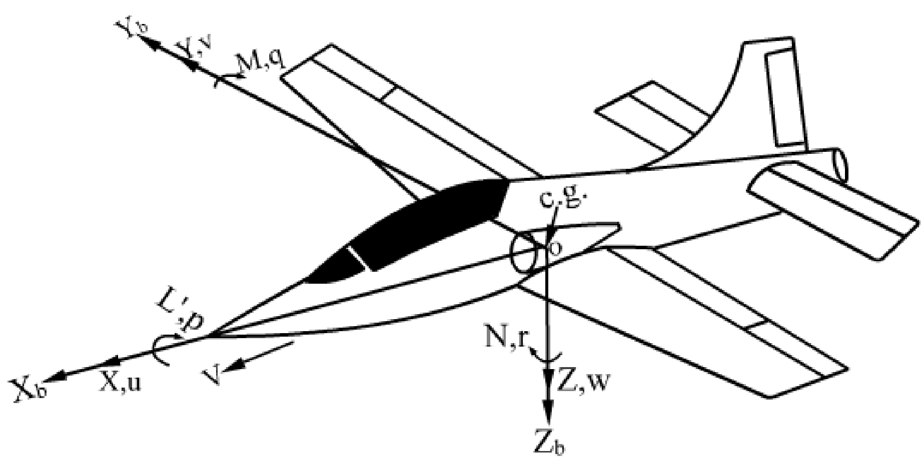


Fig.1.8 Body axes system, forces, moments and linear and angular velocities

Figure 1.8 also shows the forces and moments acting on the airplane and the components of linear and angular velocities. The quantity **V** is the velocity vector. The quantities *X, Y, Z* are the components of the resultant aerodynamic force, along OX_b, OY_b and OZ_b axes respectively. *L, M, N are the rolling moment, pitching moment and yawing moment respectively about* OX_b, OY_b *and* OZ_b; the rolling moment is denoted by L to distinguish it from lift (L) . Figure 1.8 also shows the positive directions of L, M and N. The convention is that an aerodynamic moment is taken positive in clock-wise sense when looking along the axis about which the moment is taken. *u,v,w* are the components , along OX_b, OY_b *and* OZ_b *of the velocity vector (**V**).* The angular velocity components are indicated by ***p,q,r*** or ω_x ω_y ω_z

II) Earth fixed axes system

In flight dynamics a frame of reference attached to the earth is taken as a

Newtonian frame (Fig.1.9).

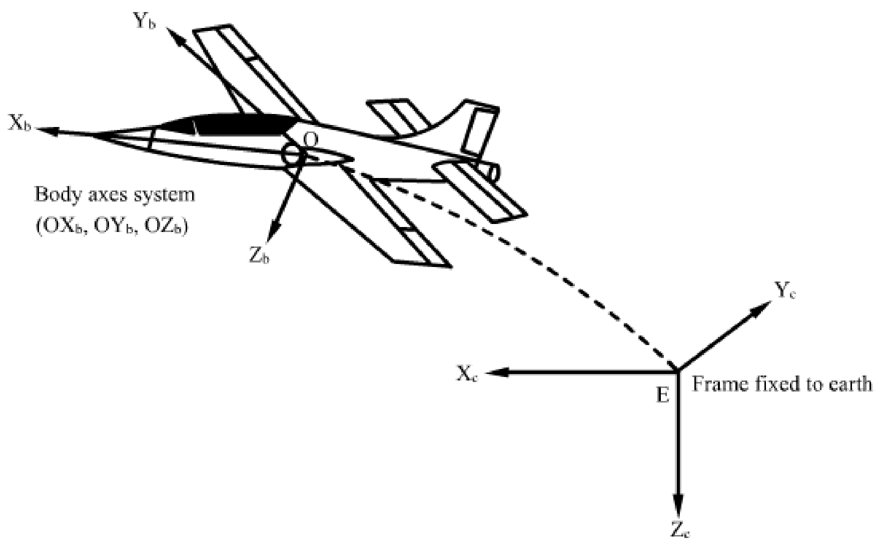


Fig.1.9 Earth fixed and body fixed co-ordinate systems

III) Attitude

In this course the airplane is treated as a rigid body. ***A rigid body has six degrees of freedom and hence, six coordinates are needed to describe the position of the airplane with respect to an earth fixed system.*** In flight dynamics, the six coordinates' employed to prescribe the position are

(a) the three coordinates describing the instantaneous position of the c.g. of the airplane with respect to the earth fixed system and

(b) the attitude of the airplane described by the angular orientations of $OX_bY_bZ_b$ system with respect to the $OX_eY_eZ_e$ system.

This is done with the help of Euler angles. To arrive at the $OX_bY_bZ_b$ system, we need to rotate the $EX_eY_eZ_e$ system through only three angles which are called ***Euler angles***.

At this stage, simpler cases are considered. When an airplane climbs along a straight line its attitude is given by the angle 'γ ' between the axis OX_b and the horizontal (Fig.1.10). When an airplane executes a turn, the projection of the OX_b axis, in the horizontal plane makes an angle Ψ with reference to fixed horizontal axis (Fig.1.11). When an airplane is banked, the axis OY_b makes an angle φ with respect to the horizontal and the axis OZ_b makes an angle φ with vertical (Fig.1.12).

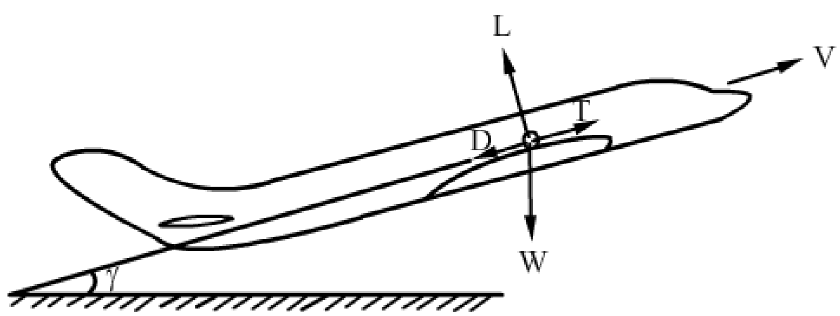


Fig.1.10 Airplane in a climb

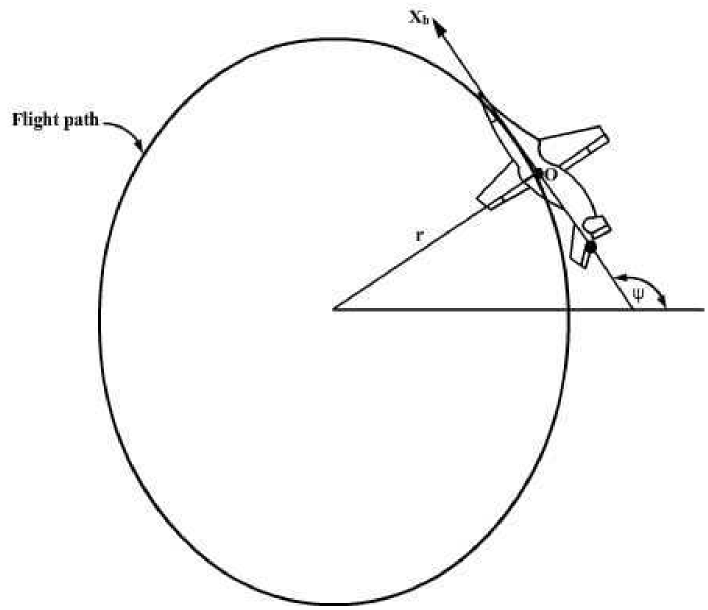


Fig.1.11 Airplane in a turn – view from top

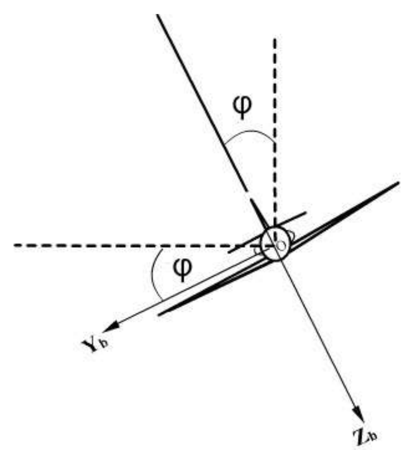


Fig.1.12 Angle of bank (φ)

IV) Flight path:

The flight path, also called the trajectory, means the path or the line along which the c.g. of the airplane moves. The tangent to this curve at a point gives the direction of flight velocity at that point on the flight path. The relative wind is in a direction opposite to that of the flight velocity.

V) Angle of attack and angle side slip

The concept of the angle of attack of an airfoil is well known. While discussing the forces acting on an airfoil, we take the chord of the airfoil as the reference line and the angle between the chord line and the relative wind is the angle of attack (α). The aerodynamic forces namely lift (L) and drag (D), produced by the airfoil, depend on the angle of attack (α) and are respectively perpendicular and parallel to relative wind direction (Fig.1.13).

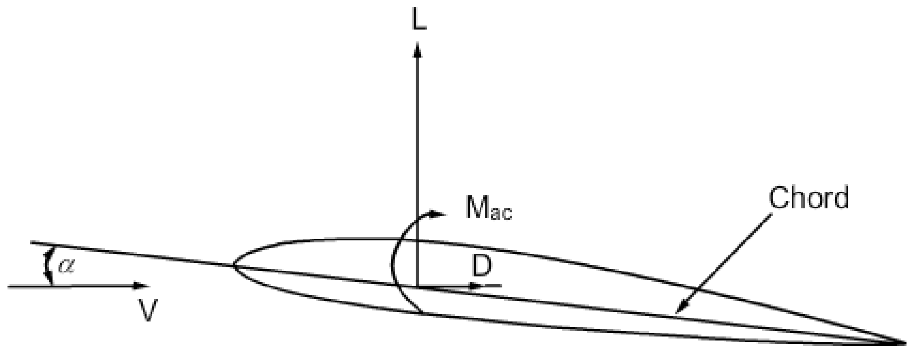


Fig.1.13 Angle of attack and forces on a airfoil

In the case of an airplane, the flight path, as mentioned earlier, is the line along which c.g. of the airplane moves. The tangent to the flight path is the direction of flight velocity (**V**). The relative wind is in a direction opposite to the flight velocity. If the flight path is confined to the plane of symmetry, then the angle of attack would be the angle between the relative wind direction and the fuselage reference line (FRL) or OX_b axis (see Fig.1.14). However, in a general case the velocity vector (**V**) will have components both along and perpendicular to the plane of symmetry. The component perpendicular to the plane of symmetry is denoted by 'v'. The projection of the velocity vector in the plane of

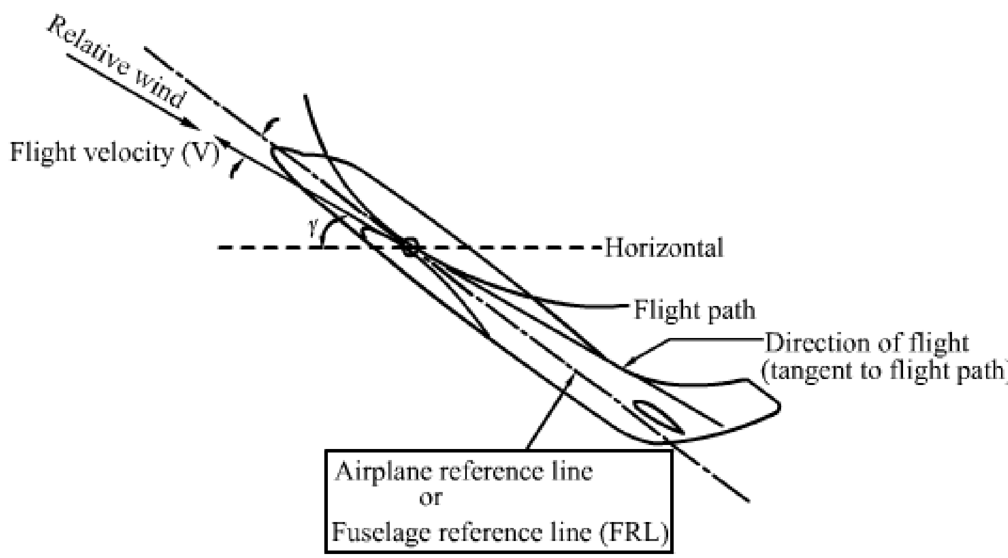


Fig.1.14 Flight path in the plane of symmetry

symmetry would have components u and w along OX_b and OZ_b axes (Fig.1.15). With this background, the angle of sideslip and angle of attack are defined below.

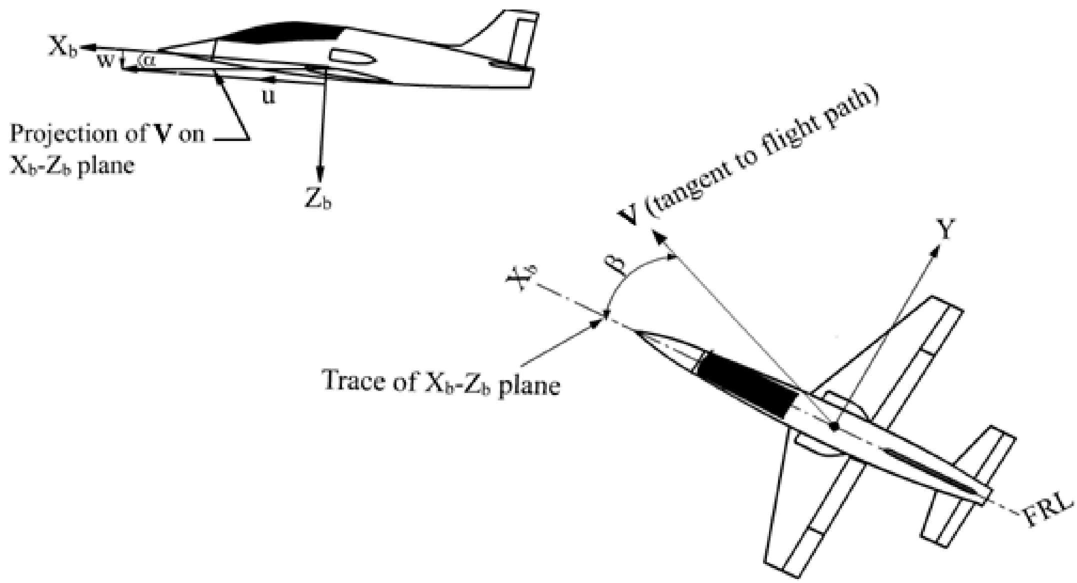


Fig.1.15 Velocity components in a general case and definition of angle of attack and sideslip

The angle of sideslip (β) is the angle between the velocity vector (**V**) and the plane of symmetry i.e.

$\beta = \sin^{-1}(v/|\mathbf{V}|)$; where $|\mathbf{V}|$ is the magnitude of **V**.

The angle of attack (α) is the angle between the projection of velocity vector (**V**) in the X_B - Z_B plane and the OX_b axis or

$$\alpha = \tan^{-1}\frac{w}{u} = \sin^{-1}\frac{w}{\sqrt{|\mathbf{V}|^2 - v^2}} = \sin^{-1}\frac{w}{\sqrt{u^2+w^2}}$$

It is easy to show that, if V denotes magnitude of the velocity (**V**) ,

then $u = V\cos\alpha\cos\beta$, $v = V\sin\beta$; $w = V\sin\alpha\cos\beta$.

1.3.5 Longitudinal and lateral stability

A relook at the stability of the pendulum, examined earlier, points out the following.

A pendulum has only one degree of freedom i.e. the rotation about the hinge. Hence, the disturbance can only be an angular displacement θ. As a result of this displacement, an unbalanced force W sin θ is created which may cause stabilizing or destabilizing moment.

On the other hand the analysis of the stability of an airplane is more complex for the following reasons.

a) An airplane in flight can move along three axes and rotate about three axes. Consequently, the disturbances can also be of various types resulting in changes in velocities along x, y and z axes and rotations about these three axes.

b) In addition to the gravitational force, an airplane is subjected to aerodynamic and propulsive forces which depend on the angle of attack and sideslip of the airplane and the linear and angular of velocities.

To make the analysis simpler, we take benefit of the fact that an airplane is symmetric about the X_b - Z_b plane (Fig.1.7).The motions along x- and z- axes and about y- axis (pitching), lie in the plane of symmetry and are called longitudinal motions. The motions along y- axis and about the x- and z- axes (rolling and yawing), which lie out of the plane of symmetry, are called lateral or asymmetric motions.

The breakup of the motion of the airplane into symmetric and asymmetric motions helps in simplifying the stability analysis. The arguments for supporting this are as follows.

A disturbance to the symmetric motions does not affect the asymmetric motions. To explain this in a better way consider an airplane in straight, level and unaccelerated flight. Let it be subjected to a disturbance in the plane of symmetry caused by either (a) a change in forward velocity or (b) a vertical velocity i.e. gust or (c) an elevator deflection. The disturbance may cause the airplane to acquire changes in u, w and q. These may cause changes in lift, drag and pitching moment. However, due to symmetry of airplane, the symmetry of the initial condition of equilibrium and the symmetry of the disturbance, the changes in lift and drag would be same on the left and right halves of the wing and the horizontal tail . Consequently, no rolling or yawing would take place i.e. the disturbances in the plane of symmetry of an airplane originally in symmetric flight, do not cause motions out of plane of symmetry.

As regards the effect of lateral disturbance on longitudinal motion, the following argument would show that the effects are very small only when the disturbance is small.

Following Ref.1.4, chapter 14, we consider that the airplane, initially in straight, level, and uncelebrated flight, is subjected to a small sideslip velocity ‘Δv’ to the right. In response to this, the airplane would tend to roll and yaw, which are motions out of plane of symmetry, but the asymmetric flow on the two wing halves and on the fuselage cause changes in pitching moment (Ref.1.5, Part II, chapter 17). Thus, a Δv produces changes in Δu, Δw and Δq. Now consider that the airplane, initially in straight level and unaccelerated flight, is subjected to sideslip Δv to the left. Besides roll and yaw, the airplane pitches but the changes are in the same direction as in the case when airplane sideslips to right. Thus, the changes in longitudinal motion Δu, Δw and Δq due to the lateral disturbance Δv do not depend on the sign of the disturbance. In other words, the changes in Δu, Δw, Δq are not proportional to Δv but to square of Δv and higher even orders of Δv. Thus if Δv is small, the changes in Δu, Δw and Δq are very small and can be ignored. But if Δv is not small, the effect on longitudinal motion would not be small.

The above arguments lead to subdivision of the stability analysis into longitudinal stability and lateral stability. The former deals with the stability of motion in the plane of symmetry and the latter deals with stability of motions out of plane of symmetry.

Remarks:

i) In static stability analysis, the perturbation in angular motions and those in the moments are predominant. Hence, in longitudinal static stability analysis the stability of motion about y-axis and in lateral static stability analysis the stability about x-and z-axes are only considered. However, in dynamic stability analysis the perturbations in linear motions are also considered.

ii) Often, the study of stability about x- axis only is called the lateral stability and that about z- axis is called the directional stability, but the two motions are interlinked and a disturbance about the z- axis produces moments about x- axis and vice versa. Hence, the lateral and directional motions are always studied together.

iii) As indicated earlier a rigid airplane has six degrees of freedom. Hence, the motion of a rigid airplane is governed by six differential equations. By separating longitudinal and lateral motions, the problem involving six equations is simplified into two problems each involving three degrees of freedom.

iv) In airplanes with features like asymmetrically swept wings and V-tail, the longitudinal and lateral motions cannot be separated. The stability analysis of these types of airplanes would require full six degrees of freedom analysis which is out of scope of the present course.

1.3.6 Control fixed and control free stability

As mentioned earlier, the airplane is treated as a rigid body for the purpose of stability analysis. This implies that the distortion of the airplane, due to aerodynamic and other loads, is small and does not change appreciably the aerodynamic characteristics of the airplane. However, the control surfaces viz. the elevator, rudder and aileron (see Fig.1.16) are movable surfaces. When they are free to move during the disturbed motion, they would bring about significant changes in the aerodynamic forces and moments in addition to those due to the disturbance. Hence, the stability of the airplane with controls fixed and controls free are analysed separately. It would be further pointed out in section 8.15 that the number of degrees of freedom increases when controls are free to move during the disturbed motion.

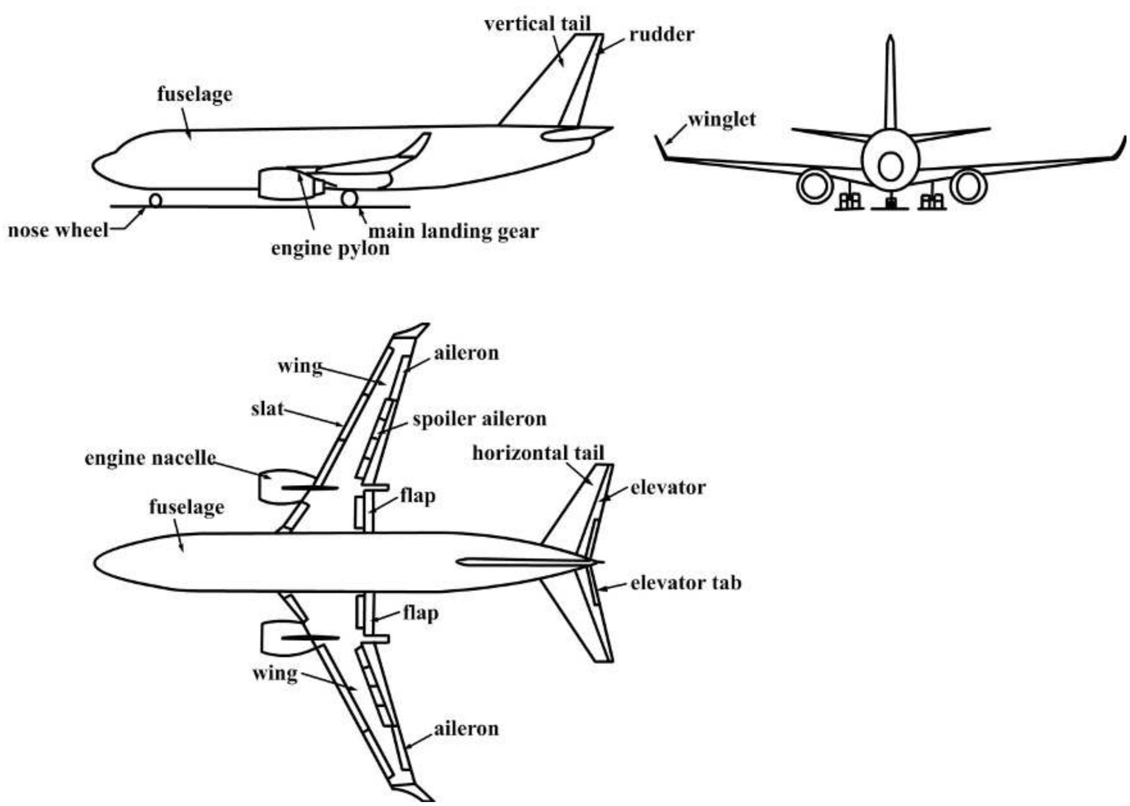


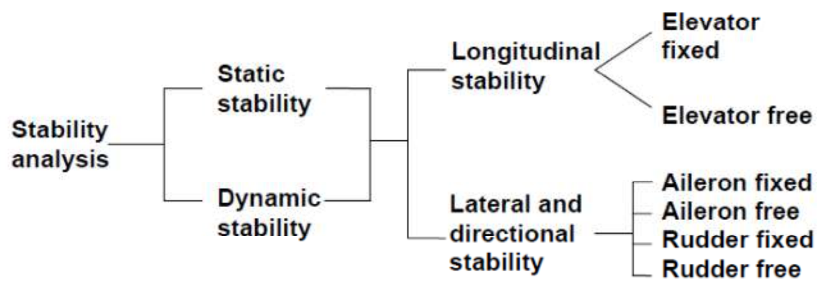
Fig.1.16 Major components including control surfaces

(aileron, elevator and rudder) of an airplane

1.3.7 Subdivisions of stability analysis

Based on the aforesaid discussion, the subject of stability analysis can be subdivided as presented in Fig.1.17.

Fig.1.17 Subdivisions of stability analysis



Remark:

Since, the movement of the elevator is controlled by stick movement, the elevator fixed and elevator free stability are also called stick-fixed and stick-free stability.

1.4 Controllability

It was pointed out earlier that for each flight condition, a definite lift coefficient and hence angle of attack is required. When the airplane is at an angle of attack the components of the airplane like wing, fuselage and tail would also be at angles of attack, and produce lift and drag which would cause pitching moments about c.g.. The sum of these pitching moments will be counter-balanced by the elevator. Hence, a suitable elevator deflection is needed for each angle of attack. Similarly, suitable rudder and aileron deflections are also needed to balance rolling moment and yawing moment during the flight. In this background, the range of speeds at which controlled flight is possible and the rapidity with which a desired attitude can be achieved, are the important factors that determine the controllability of an airplane. In general, the term controllability can be defined as the influence which the pilot or the controlling agency can exert on the equilibrium state of the airplane; this state is characterized by the variables u, v, w, p, q and r.

1.5 General remarks

In this section a few remarks are presented to supplement the description given in the earlier sections.

1.5.1 Examples of stability in day-to-day life

Examples of systems displaying different types of stability can be found in many devices in common use. For example, we can observe three different types of doors in offices. The most common type of door is neutrally stable, i.e. once opened; it remains open till someone closes it.The second type, the pair of doors with springs at the hinges, once opened and left, return to the closed position after performing a damped oscillation. The third type, the door with a hydraulic damper, when opened, returns to the closed position without oscillating and displays subsidence.

1.5.2 Airplane stability depends on flight condition

Unstable systems are difficult to observe as most of the practical systems are designed to be stable. However, systems need not be stable under all situations, e.g. an airplane that is stable in steady level flight may be unstable during an inverted flight condition. Further, an airplane which is stable at high speeds may show instability at low speeds. Hence, the stability of the airplane needs to be examined under various flight conditions.

1.5.3 Stability and controllability are not the same

Stability and controllability must be clearly distinguished for each other. The former is the ability to return to the equilibrium states after a small disturbance, whereas the latter is the ability to change from one equilibrium state to another. Therefore, a very stable airplane will resist changes in its attitude and hence, will be difficult to control. Accordingly, military airplanes, for which rapid maneuverability is one of the requirements, have lower levels of stability than civil airplanes.

1.5.4 Stability is desirable but not necessary for piloted airplanes

Stability is desirable but not a necessary for piloted airplanes. In these types of airplanes, neutral stability or a slight instability under some conditions can be tolerated if the disturbance does not grow rapidly and the pilot has enough time
to correct the situation. However, an unstable airplane requires constant attention and is a source of fatigue to the pilot.

1.5.5 Small disturbance analysis of stability

In conventional stability analysis we consider the forces and moments brought about by the disturbance as transient and small. This simplification converts the non-linear dynamic stability equations into a set of linear equations (see chapter 5).

1.5.6 Rigorous definitions of terms

The discussion on stability presented above, is somewhat simplified as this is an introductory course. Reference 1.6 , chapter 15 may be referred to for mathematical definitions of the terms like
(a) system, (b) equilibrium state (c) stability, (d) asymptotic stability , (e) asymptotic stability in large and (f) instability .

1.6 Course content

The subject matter here is divided into the following topics.
Chapter 1& 2. Longitudinal stick-fixed stability and control

Chapter 2. Longitudinal stick-free static stability and control

Chapter 4. Longitudinal static stability and control – effect of acceleration

Chapter 5. Directional static stability and control

Chapter 6.. Lateral static stability and control

Chapter 7. Dynamic stability analysis – I – Equations of motion and estimation of stability derivatives
Chapter 8. Dynamic stability analysis – II – Longitudinal motion

Chapter 9. Dynamic stability analysis – III – Lateral motion

Chapter10. Miscellaneous topics – stability after stall, automatic control and response.
Sample question paper –Hints for solutions

Sample question paper –Model answers

Appendix 'C' presents drag polar, stability derivatives and characteristic roots for a jet airplane.

1.7 Background expected

It is expected that the student has undergone course on Flight mechanics-I

i.e. airplane performance which calls for background of

(a) vectors, (b) rigid body dynamics (c) aerodynamics and (d) airplane engines.

Remark:

In addition to Refs.1.1 to 1.6 mentioned earlier, references 1.7 to 1.13 may be consulted for further information on stability and control analysis.

2 & 3. Longitudinal stick-fixed stability and control

Keywords : Criteria for longitudinal static stability and control ; contributions of wing, horizontal tail, fuselage and power to pitching moment coefficient (C_{mcg}) and its derivative with respect to angle of attack ($C_{m\alpha}$) ; stick-fixed neutral point and static margin ; elevator angle for trim; limitations on forward and rearward movements of c.g. ; determination of neutral point from flight tests.

2.1 Introduction

2.1.1 Equilibrium state during flight in the plane of symmetry

2.1.2 Mean aerodynamic chord

2.1.3 Criteria for longitudinal control and trim in pitch

2.1.4 Criterion for longitudinal static stability

2.1.5 Alternate explanation of criterion for longitudinal static stability

2.1.6 Desirable values of C_{m0} and $C_{m\alpha}$

2.1.7 Effect of elevator deflection on C_{mcg} vrs α curve

2.1.8 C_{mcg} expressed as function of C_L

2.2 C_{mcg} and $C_{m\alpha}$ as sum of the contributions of various component

2.3 Contributions of wing to C_{mcg} and $C_{m\alpha}$

2.3.1 Correction to $C_{m\alpha w}$ for effects of horizontal components of lift and drag – secondary effect of wing location on static stability

2.4 Contributions of horizontal tail to C_{mcg} and $C_{m\alpha}$

2.4.1 Conventional tail, canard configuration and tailless configuration

2.4.2 Effect of downwash due to wing on angle of attack of tail

2.4.3 Interference effect on dynamic pressure over tail

2.4.4 Expression for C_{mcgt}

2.4.5 Estimation of C_{Lt}

2.4.6 Revised expression for C_{mcgt}

2.4.7 $C_{m\alpha t}$ in stick-fixed case

2.1 Introduction

For a gradual development of the stability and control analysis, the subject has been subdivided into various topics (see subsection 1.3.7 and Fig.1.17). This chapter deals with longitudinal static stability and control in stick-fixed case. The following three items from chapter 1 may be recalled.

(a)In static stability analysis, the forces and moments brought about as a result of the disturbance are considered to examine whether moments tend to bring the airplane back to the equilibrium state or not.

(b)The longitudinal stability analysis deals with the motions in the plane of symmetry i.e. along x- and z- axes and about y-axis.

(c) By stick-fixed case, we imply that even after the disturbance is applied, the control stick is held fixed or the control surface maintains its deflection as in the undisturbed state.

2.1.1 Equilibrium state during flight in the plane of symmetry

To analyze the stability, we must consider that the airplane is initially in equilibrium state i.e. it is moving with constant speed along a straight line. If we consider the steady level flight, the conditions for equilibrium are:

$$T - D = 0 \tag{2.1}$$

$$L - W = 0 \tag{2.2}$$

$$M_{cg} = 0 \tag{2.3}$$

These equations imply that:

(a) the thrust must balance the drag by proper setting of the engine throttle,

(b) the lift must balance the weight by proper choice of the lift coefficient at the chosen speed and altitude of flight and

(c) the pitching moment produced by the wing, the fuselage, the tail and other components must be counterbalanced by the moment produced by the elevator.

Thus, longitudinal control implies the ability to bring M_{cg} to zero by suitable control deflection. In order that this is achieved under different flight conditions, the elevator must have sufficient area and adequate range of deflections. For longitudinal static stability, we need to primarily examine the rotation about the y-axis. The moment about the y-axis is the pitching moment (M or M_{cg}). A nose up moment is taken as positive (Fig.2.1).

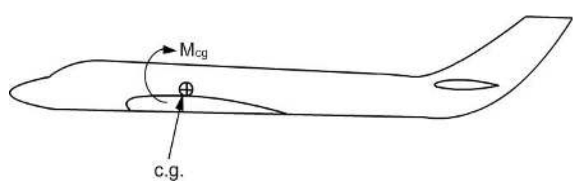


Fig.2.1 Convention for pitching moment

It is convenient to work in terms of pitching moment coefficient (C_{mcg}), which is

defined as:

$$C_{mcg} = \frac{M_{cg}}{\frac{1}{2}\rho V^2 S\bar{c}} \tag{2.4}$$

where, ρ = ambient density; V = flight velocity; S = wing plan form area;

$\bar{c}$ = mean aerodynamic chord of wing.

2.1.2 Mean aerodynamic chord

It may be recalled that the mean aerodynamic chord $\bar{c}$ is defined a

$$\bar{c} = \frac{1}{S}\int_{-b/2}^{b/2} c^2\, dy \tag{2.5}$$

where, c is the local chord of wing (Fig.2.2) and b is the wing span.

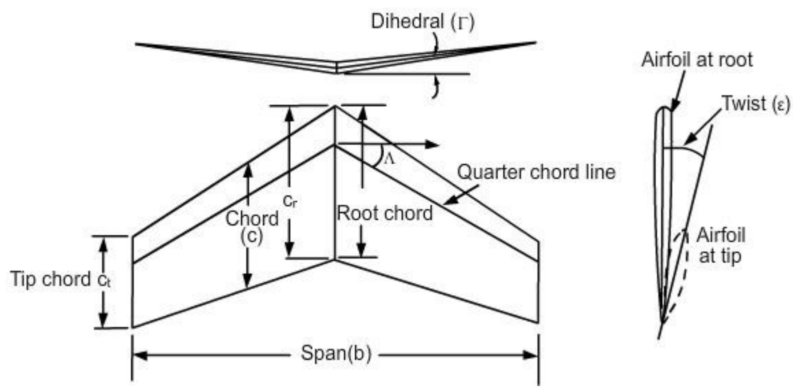


Fig.2.2 Geometric parameters of a wing

For a trapezoidal wing the mean aerodynamic chord is given by the following expression, the derivation is left as an exercise to the reader.

$$\bar{c} = \frac{2}{3}\frac{c_r(1+\lambda+\lambda^2)}{1+\lambda}$$

where, λ = taper ratio = c_t/c_r . (2.5a)

2.1.3 Criterion for longitudinal control and trim in pitch

For equilibrium,

$C_{mcg} = 0$ (2.6)

When C_{mcg} is made zero by proper control deflection, the airplane is said to be trimmed in pitch.

2.1.4 Criterion for longitudinal static stability

The criterion for longitudinal static stability is that when an airplane is disturbed in the plane of symmetry, it has a tendency to return to its equilibrium state. In longitudinal static stability analysis, the effects of perturbations Δu and Δw are negligible. The change in the angle of attack ($\Delta\alpha$) is considered as the perturbation. Its effect on change in pitching moment, ΔM_{cg}, is examined to assess the longitudinal static stability. The change in the angle of attack of the airplane could be due to:

(a) the airplane encountering a vertical gust of velocity (V_{gu}) or

(b) the pilot deflecting the elevator by a small angle causing a moment and then bringing the elevator to its position in the undisturbed flight .

The gust would change the direction of relative wind and cause a change in the angle of attack (Fig.2.3) given by:

$$\Delta\alpha = V_{gu} / V$$

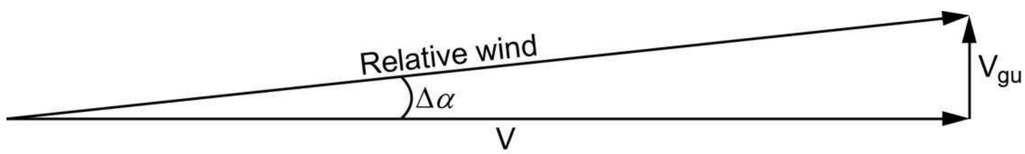


Fig.2.3 change in angle of attack due to gust

The deflection of elevator by the pilot will also lead to change in the angle of attack. Note that the convention for $\Delta\alpha$ is the same as that for the angle of attack i.e. measured from the relative wind towards the fuselage reference line (FRL) and taken positive clock wise (Fig.2.4). With the above conventions for M_{cg} and $\Delta\alpha$, if the airplane is to have static stability, then in response to a positive $\Delta\alpha$ caused by the disturbance, the airplane should produce a ΔM_{cg} which is negative. Similarly, a disturbance causing negative $\Delta\alpha$ should result in positive ΔM_{cg} or ($dM_{cg}/ d\alpha$) should be negative.

2.1.5 Alternate explanation of criterion for a longitudinal static stability

The above argument can be explained in an alternative manner. Consider that the airplane is flying in level flight at angle of attack α i.e. the c.g. moves along a horizontal line and the fuselage reference line (FRL) makes an angle α to the flight direction (Fig.2.4). Now, imagine that the airplane is disturbed and acquires an additional angle of $\Delta\alpha$ i.e. its angle of attack becomes ($\alpha+\Delta\alpha$). The airplane in the changed attitude is shown by dotted lines in Fig.2.4. Now, if the airplane has static stability, then it should produce ΔM_{cg} such that the airplane returns to the original angle of attack i.e. a disturbance which causes positive $\Delta\alpha$, should result in negative ΔM_{cg} . Similarly, a disturbance that causes negative

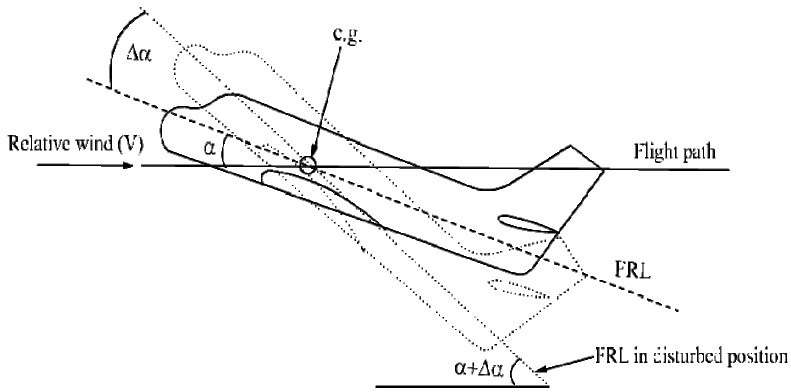


Fig.2.4 Airplane in disturbed position

$\Delta\alpha$ should be accompanied by positive ΔM_{cg} . This again means that

$(dM_{cg} / d\alpha)$ or $(dC_{mcg} /d\alpha)$ should be negative. If $(dC_{mcg} / d\alpha)$ is zero then the airplane has no tendency to come back and is neutrally stable. If $(dC_{mcg} / d\alpha)$ is positive then the moment produced as a result of positive $\Delta\alpha$ is also positive and would take the airplane to increased $\Delta\alpha$. This means that the airplane is unstable.

Thus, the criterion for longitudinal static stability is:

$(dM_{cg}/d\alpha)$ < 0 for static stability

> 0 for instability (2.7)

$= 0$ for neutral stability

Or, $dC_{mcg} /d\alpha$ or $C_{m\alpha}$ < 0 for static stability

> 0 for instability (2.8)

$= 0$ for neutral stability

Remark:

The angle $\Delta\alpha$ in Fig.2.4 is portrayed big for the sake of clarity. In static stability analysis $\Delta\alpha$ would be small.

2.1.6 Desirable values of C_{m0} and $C_{m\alpha}$

Figure 2.5 shows C_{mcg} vs α curves for two airplanes A and B. Both configurations A and B are in trim (i.e. $C_{mcg} = 0$) at point P without deflection of control surface (elevator). However, from stability criterion given in Eq.(2.8), the configuration A, with $C_{m\alpha} < 0$, is stable and the configuration B, with $C_{m\alpha} > 0$, is unstable. This figure also shows that for airplane A the value of C_{m0} (i.e. value of C_{mcg} when α is zero) is positive. These factors indicate that, for an airplane to be both stable and give trim at realistic values of C_L , requires that :

(a) C_{m0} should be positive and

(b) $C_{m\alpha}$ should be negative.

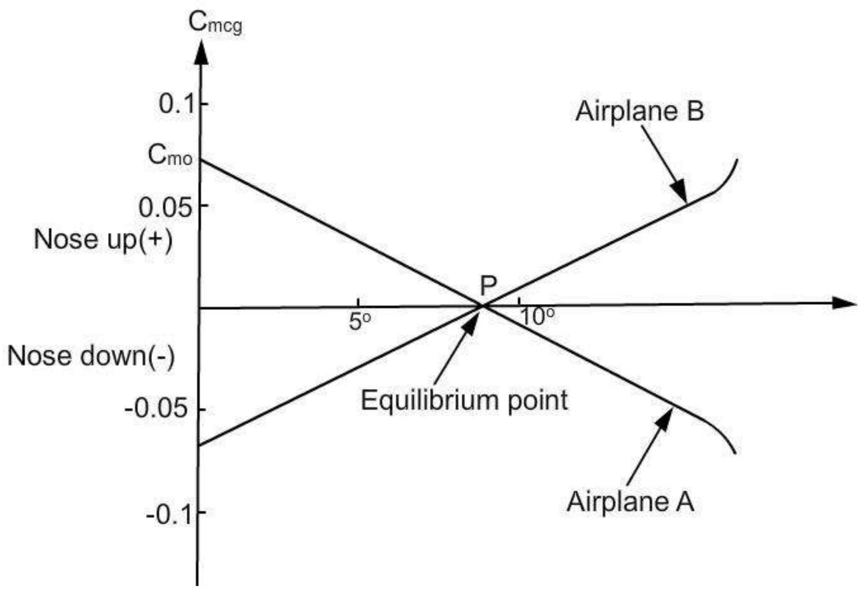


Fig.2.5 C_{mcg} vs α variations for stable and unstable airplanes

2.1.7 Effect of elevator deflection on C_{mcg} vs α curve

When an elevator is deflected it produces a moment about c.g. Then the value of C_{m0} of the airplane changes and the C_{mcg} vs α curve is shifted (Fig.2.6). However, $C_{m\alpha}$ does not change due to the elevator deflection and the slope of the curve is same as that with zero elevator deflection (see section 2.12.3 and example 2.7). This figure also indicates that elevator deflection brings about change in the value of α at which C_{mcg} is zero or the airplane is in trim. It may be pointed out that the elevator deflection is denoted by δ_e and downward deflection of elevator is taken positive (see section 2.4.5 for further details).

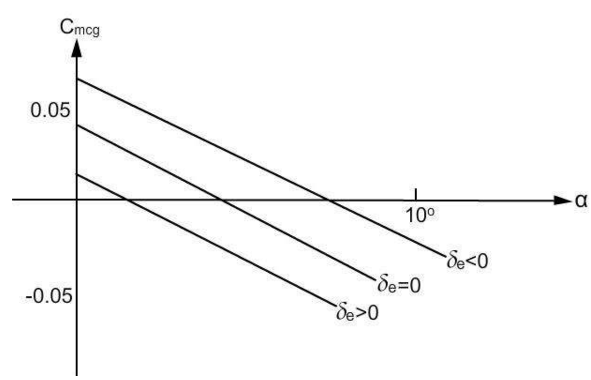


Fig.2.6 Effect of elevator deflection on trim

2.1.8 C_{mcg} expressed as function of C_L

When the angle of attack (α) is not near stalling angle, the C_L vs α curve of the airplane is nearly linear (Fig.2.7). In this situation C_{mcg} can be plotted as functions of C_L and dC_m/dC_L can be used as criterion for static stability instead of
$C_{m\alpha}$ .This was the practice in older literature, on stability and control e.g. Ref.1.7.

Noting that

$$\frac{dC_m}{d\alpha} = \frac{dC_m}{dC_L}\frac{dC_L}{d\alpha} = C_{L\alpha}\frac{dC_m}{dC_L}$$

yields:

$$\frac{dC_m}{dC_L} = \frac{C_{m\alpha}}{C_{L\alpha}} \qquad (2.9)$$

When C_L vs α is linear, the longitudinal static stability criterion is:

$d\,C_m / d\,C_L$ < 0 for static stability

> 0 for instability (2.10)

= 0 for neutral stability

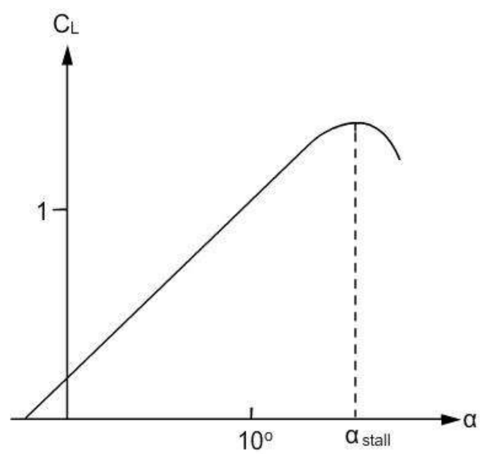


Fig.2.7 C_L vs α

2.2 C_{mcg} and $C_{m\alpha}$ expressed as sum of the contributions of various components of the airplane

Using wind tunnel tests on a model of an airplane or by Computational Fluid Dynamics (CFD), the C_{mcg} vs α curve for the entire airplane can be obtained. However, CFD has not yet advanced enough to give accurate values of the moments and these computations are not inexpensive. Wind tunnel tests are very expensive and are resorted to only at the later stages of airplane design. Hence, the usual practice to obtain the C_{mcg} vs α curve is to add the contributions of major components of the airplane and at the same time take into account the interference effects. The contributions of individual components are based on the wind tunnel data or the analyses available in literature. References 1.1,1.8,1.9, 1.12, 2.1 and 2.2 are some of the sources of data.

The contributions to C_{mcg} and $C_{m\alpha}$ are due to the wing, the fuselage, the power plant and the horizontal tail. Figure 2.8 shows the forces and moments produced by the wing and the horizontal tail. The contributions of fuselage, nacelle and the power plant are shown as moments about c.g. and denoted by $M_{f,n,p}$. ***The fuselage reference line is denoted by FRL***. It may be recalled that the angle of attack (α) of the airplane is the angle between free stream velocity (V) and FRL. The c.g. of the airplane is also shown in the figure. The wing is represented by its mean aerodynamic chord (m.a.c.). It is set at an angle of incidence i_w to the FRL.

Hence, the angle of attack of wing (α_w) is $\alpha + i_w$. Following the usual practice, the lift of the wing (L_W) is placed at the aerodynamic centre of the wing (a.c.) along with a pitching moment (M_{acw}). The drag of the wing (D_w) is also taken to act at the aerodynamic centre of the wing. The wing a.c. is located at a distance x_{ac} from the leading edge of the m.a.c. The airplane c.g. is at a distance x_{cg} from the leading edge of the m.a.c.

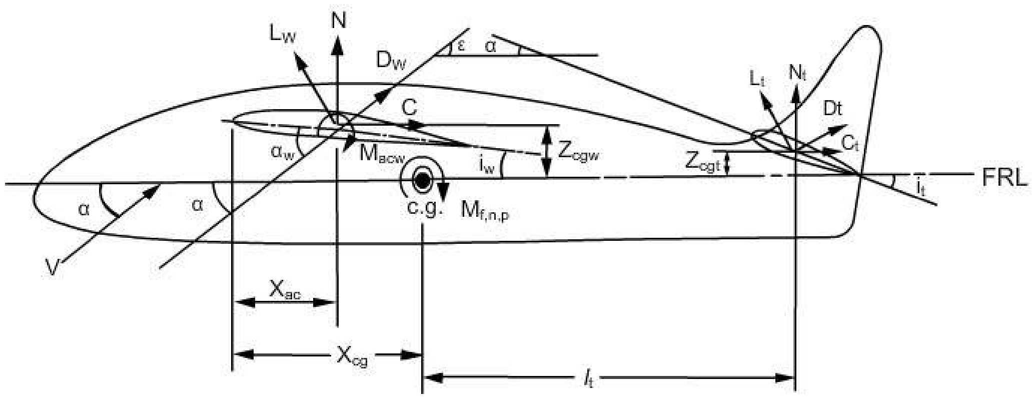


Fig.2.8 Contributions of major components to C_{mcg}

The horizontal tail is also represented by its mean aerodynamic chord. The aerodynamic centre of the tail is located at a distance l_t behind the c.g. The tail is mounted at an angle i_t with respect to the FRL. The lift, drag and pitching moment due to the tail are L_t, D_t and M_{act} respectively. As the air flows past the wing, it experiences a downwash ε which is shown schematically in Fig.2.8. Owing to this the angle of attack of the horizontal tail would be ($\alpha + i_t - \varepsilon$). Further, due to the interference effects the tail would experience a dynamic pressure different from the free stream dynamic pressure. These aspects will be elaborated in section 2.4.2 and 2.4.3. With this background the pitching moment

about the c.g. can be expressed as:

$$M_{cg} = (M_{cg})_w + (M_{cg})_f + (M_{cg})_n + (M_{cg})_p + (M_{cg})_t \qquad (2.11)$$

$$C_{mcg} = \frac{M_{cg}}{\frac{1}{2}\rho V^2 S\bar{c}} = (C_{mcg})_w + (C_{mcg})_{f,n,p} + (C_{mcg})_t \qquad (2.12)$$

$$C_{m\alpha} = (C_{m\alpha})_w + (C_{m\alpha})_{f,n,p} + (C_{m\alpha})_t \qquad (2.13)$$

Note:

(i) For convenience the derivative of C_{mcg} with α is denoted as $C_{m\alpha}$.

(ii) In Fig.2.8 the angle 'i_t' is shown positive for the sake of indicating the notation; generally 'i_t' is negative. The contributions to C_{mcg} the next four sections.
and $C_{m\alpha}$ of the individual components are described in

2.3 Contributions of wing to C_{mcg} and $C_{m\alpha}$

Figure 2.9 schematically shows the forces (lift, L_w and drag, D_w) and the moment (M_{acw}) due to the wing and the relative locations of the c.g. of the airplane and the aerodynamic centre of the wing.
The following may be recalled / noted.

i) The angle of attack of the airplane is the angle between the relative wind and the fuselage reference line (FRL). This angle is denoted by α.
ii) The wing is represented by its mean aerodynamic chord (m.a.c.).

iii) The wing is set at an angle i_w to the FRL. This is done so that the fuselage is horizontal during cruising flight. Thus, $\alpha_w = \alpha + i_w$ or $\alpha = \alpha_w - i_w$.
iv) x_{ac} is the distance of the a.c. from the leading edge of the m.a.c..
v) x_{cg} is the distance of the c.g. from the leading edge of the m.a.c..
vi)Z_{cgw} is the distance of the a.c. below c.g.

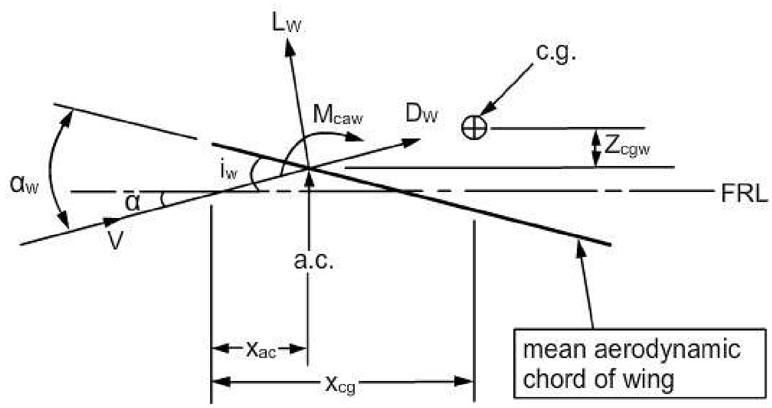


Fig.2.9 Wing contribution

Taking moment about c.g., gives the contribution of wing (M_{cgw}) to the moment about c.g as:

$M_{cgw} = L_w \cos(\alpha_w - i_w)[x_{cg} - x_{ac}] + D_w \sin(\alpha_w - i_w)[x_{cg} - x_{ac}] + L_w \sin(\alpha_w - i_w) Z_{cgw} - D_w \cos(\alpha_w - i_w) Z_{cgw} + M_{acw}$

(2.14)

Noting that,

$$C_{mcgw} = \frac{M_{cgw}}{\frac{1}{2}\rho V^2 S \bar{c}}; \; C_{Lw} = \frac{L_w}{\frac{1}{2}\rho V^2 S}; \; C_{Dw} = \frac{D_w}{\frac{1}{2}\rho V^2 S}; \; C_{macw} = \frac{M_{acw}}{\frac{1}{2}\rho V^2 S \bar{c}}, \quad (2.15)$$

yields:

$$C_{mcgw} = C_{Lw}\cos(\alpha_w - i_w)\left[\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right] + C_{Dw}\sin(\alpha_w - i_w)\left[\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right]$$
$$+ C_{Lw}\sin(\alpha_w - i_w)\frac{Z_{cgw}}{\bar{c}} - C_{Dw}\cos(\alpha_w - i_w)\frac{Z_{cgw}}{\bar{c}} + C_{macw} \quad (2.16)$$

Remark:

$(\alpha_w - i_w)$ is generally less than 10^0.Hence, $\cos(\alpha_w - i_w) \approx 1$; and $\sin(\alpha_w - i_w) \approx (\alpha_w - i_w)$.
Further $C_L >> C_D$.
Neglecting the products of small quantities, Eq.(2.16) reduces to:

$$C_{mcgw} = C_{macw} + C_{Lw}\left[\frac{X_{cg}}{\bar{c}} - \frac{X_{ac}}{\bar{c}}\right] \quad 2.17$$

Now,

$$\begin{aligned} C_{Lw} &= C_{L\alpha w}(\alpha_w - \alpha_{0Lw}) \\ &= C_{L\alpha w}(\alpha + i_w - \alpha_{0Lw}) \\ &= C_{L\alpha w}(i_w - \alpha_{0Lw}) + C_{L\alpha w}\alpha \\ &= C_{L0w} + C_{L\alpha w}\alpha \end{aligned} \quad 2.18$$

where, α_{0Lw} is the zero lift angle of the wing and

$C_{L0w} = C_{L\alpha w}(i_w - \alpha_{0Lw})$

Hence,

$$C_{mcgw} = C_{macw} + C_{L0w}\left[\frac{X_{cg}}{c} - \frac{X_{ac}}{c}\right] + C_{L\alpha w}\,\alpha\left[\frac{X_{cg}}{c} - \frac{X_{ac}}{c}\right] \qquad 2.19$$

Differentiating with respect to α , gives the contribution of wing to $C_{m\alpha}$ as :

$$C_{m\alpha w} = C_{L\alpha w}\left[\frac{X_{cg}}{c} - \frac{X_{ac}}{c}\right] \qquad 2.20$$

Remark:

The contribution of wing (C_{mcgw}) as approximately calculated above and given by Eq.(2.19) is linear with α. When the a.c. is ahead of c.g., the term $\left[\frac{X_{cg}}{c} - \frac{X_{ac}}{c}\right]$ is positive and consequently $C_{m\alpha w}$ is positive (Eq.2.20). Since, $C_{m\alpha}$ should be negative for static stability, a positive contribution to $C_{m\alpha}$ is called destabilizing contribution. When the a.c. is ahead of c.g. the wing contribution is destabilizing. Figure 2.10 shows C_{mcgw} vs α in this case.

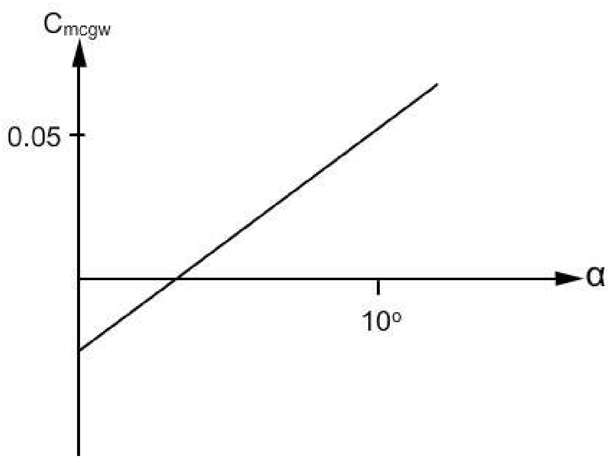


Fig.2.10 Approximate contribution of wing to C_{mcg}

2.3.1 Correction to $C_{m\alpha w}$ for effects of horizontal components of lift and drag – secondary effect of wing location on static stability

In the simplified analysis for the contribution of wing to C_{mcg} , the contributions of the horizontal components of lift and drag to the moment about c.g. , have been ignored (compare Eqs. 2.16 and 2.17).Let, the neglected terms be denoted by M_{cgwh}. Equation (2.14) gives the following expression for M_{mcgwh}

$$M_{cgwh} = L_w \sin(\alpha_w - i_w) Z_{cgw} - D_w \cos(\alpha_w - i_w) Z_{cgw} \qquad 2.21$$

Dividing by ½ $\rho V^2 S \bar{c}$ and noting that

$$\cos(\alpha_w - i_w) \approx 1$$

yields :

$$C_{mcgwh} = [C_{Lw}\sin(\alpha_w - i_w) - C_{Dw}]\frac{Z_{cgw}}{\bar{c}};$$

Differentiating Eq. 2.22

$$C_{m\alpha wh} = [\frac{dC_{Lw}}{d\alpha}\sin(\alpha_w - i_w) + C_{Lw}\cos(\alpha_w - i_w) - \frac{dC_{Dw}}{d\alpha}]\frac{Z_{cgw}}{\bar{c}}$$

$$\text{Now}, \frac{dC_{Lw}}{d\alpha}\sin(\alpha_w - i_w) \approx C_{L\alpha w}(\alpha_w - i_w)$$

2.23

$C_{L\alpha w}(\alpha_w - i_w) = C_{L\alpha w}(\alpha_w - \alpha_{0L}) - C_{L\alpha w}(i_w - \alpha_{0L}) = C_{Lw} - C_{L0w}$

Further,

$$C_{Lw}\cos(\alpha_w - i_w) \approx C_{Lw}$$

And

$$\frac{dC_{Dw}}{d\alpha} = \frac{dC_{Dw}}{dC_L}\frac{dC_L}{d\alpha} = C_{L\alpha w}\frac{dC_{Dw}}{dC_L}$$

2.24

Thus,

$$C_{m\alpha wh} = [2C_{Lw} - C_{L0w} - C_{L\alpha w}\frac{dC_{Dw}}{dC_L}]\frac{Z_{cgw}}{\bar{c}}$$ 2.25

The drag polar of the wing can be assumed as :

$$C_{Dw} = C_{D0w} + \frac{C_{Lw}^2}{\pi A e}$$

Then,

$$\frac{dC_{Dw}}{dC_L} = \frac{2C_{Lw}}{\pi A e}$$

Substituting this in Eq.(2.25) yields:

$$C_{m\alpha wh} = [2C_{Lw} - C_{L0w} - C_{L\alpha w}\frac{2C_{Lw}}{\pi Ae}]\frac{Z_{cgw}}{\bar{c}}$$

$$C_{m\alpha wh} = [2C_{Lw}\{1 - \frac{C_{L\alpha w}}{\pi Ae}\} - C_{L0w}]\frac{Z_{cgw}}{\bar{c}} \qquad (2.26)$$

The term $[1 - (2C_{L\alpha w} / \pi Ae)]$ is generally positive. This can be seen as follows. An approximate expression for $C_{L\alpha w}$ is:

$$C_{L\alpha w} = 2\pi\frac{A}{A+2}\text{ ; A = Aspect ratio of wing.}$$

Hence

$$\frac{C_{L\alpha w}}{\pi Ae} = 2\pi\frac{A}{A+2}\,\frac{1}{\pi Ae} = \frac{2}{(A+2)e}, \qquad (2.27)$$

$2/\{(A+2)e\}$ is less than 1 for typical values of A and e.
Further, for low wing aircraft, where the a.c of the wing is below c.g., the term $Z_{cgw}/\bar{c}$ is positive (Fig.2.9) . Hence, $C_{m\alpha wh}$ as given by Eq.(2.26) is positive or destabilizing (Fig.2.11). For high wing aircraft, $Z_{cgw}/\bar{c}$ is negative consequently $C_{m\alpha wh}$ is negative and hence stabilizing (Fig.2.11).

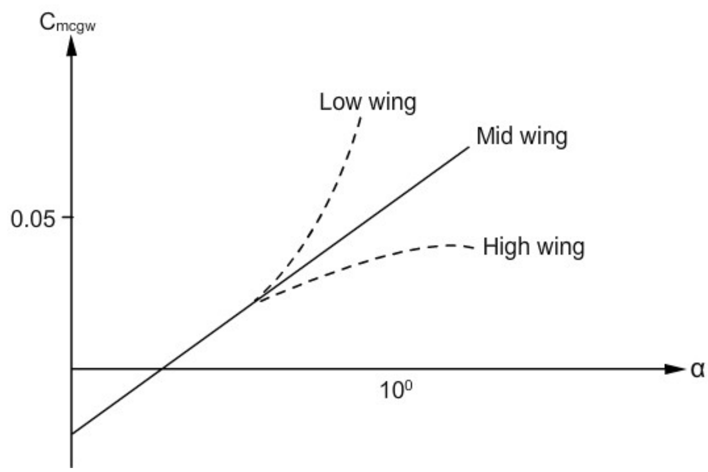


Fig.2.11 Effect of wing location on C_{mcgw}

An important aspect of the above derivation may be pointed out here. The expression for $C_{m\alpha wh}$ involves C_L or the slope of C_{mcgw} vs α curve depends on C_L or α (see example 2.3) . Hence, C_{mcgw} become slightly non-linear. The usual practice, is to ignore the contributions of the horizontal components to $C_{m\alpha w}$. However, the following aspects may be pointed out. (a) A high wing configuration is slightly more stable than a mid-wing configuration. A low wing configuration is slightly less stable than the mid-wing configuration. (b) In the simpler analysis the C_{mcgw} vs α curve is treated as straight line but the C_{mcg} vs α curves, obtained from flight tests on airplanes, are found to be slightly non-linear. One of the reasons for the non-linearity in actual curves is the term M_{egwh}.

Example 2.1

Given a rectangular wing of aspect ratio 6 and area 55.8 m^2. The wing section employed is an NACA 4412 airfoil with aerodynamic centre at 0.24 c and C_{mac} = -0.088.The c.g. of the wing lies on the wing chord, but 15 cm ahead of the a.c. Calculate the following.

(a) The lift coefficient for which the wing would be in equilibrium ($C_{mcg} = 0$). Is this lift coefficient useful? Is the equilibrium statically stable?

(b) Calculate the position of c.g. for equilibrium at $C_L = 0.4$. Is this equilibrium statically stable?

Example 2.2

If the wing given example 2.1 is rebuilt maintaining the same planform, but using reflex cambered airfoil section such that C_{mac} = 0.02, with the a.c. still at 0.24 c . Calculate the c.g. position for equilibrium at $C_L = 0.4$. Is this equilibrium

statically stable?

Example 2.3

An airplane is equipped with a wing of aspect ratio 6 ($C_{l\alpha w}$ = 0.095) and span efficiency factor e of 0.78, with an airfoil section giving C_{mac} = 0.02. Calculate, for C_L between 0 and 1.2, the pitching moment coefficient of the wing about the c.g. which is located 0.05 c ahead of a.c. and 0.06 c under a.c.. Repeat the calculations when chord wise force component is neglected. Assume

$$C_{D0w} = 0.008,\ \alpha_{oLw} = 1^0,\ i_w = 5^0.$$

2.4. Contributions of horizontal tail to C_{mcg} and $C_{m\alpha}$

In this section the contributions of horizontal tail to C_{mcg} and $C_{m\alpha}$ and the related aspects are dealt with. In this chapter and in chapters 3 and 4, the horizontal tail is simply referred to as tail.

2.4.1 Conventional tail, canard configuration and tailless configuration

A horizontal tail, as explained in this section, provides stability about y- axis. Hence, it is called horizontal stabilizer. When the horizontal stabilizer is behind the wing it is called conventional tail configuration. It is also explained, later in subsection 2.12.3, that for achieving equilibrium with conventional tail configuration, the lift on the tail is generally in the downward direction. This necessicitates that the lift produced by the wing has not only to balance the airplane weight but also the negative lift on the tail. This can be avoided if a control surface is located ahead of the wing. Such a configuration is called canard (see Wright flyer in Fig.1.1). It may be added that a canard, being ahead of c.g., has destabilizing contribution to $C_{m\alpha}$. There are airplanes which neither have a horizontal tail nor a canard surface. In this case the airplane is called "Tailless configuration" (See Concorde in Fig.1.4a). Now, the conventional tail configuration is considered in detail. The contribution of the tail depends on L_t, D_t and M_{act} (Fig. 2.8). However, these quantities depend on the angle of attack of the tail and the dynamic pressure experienced by the tail. The angle of attack of the tail is not just $(\alpha+i_t)$. The downwash behind the wing affects the angle at which the flow reaches the tail. Further, the wake of the wing and the boundary layer on the fuselage render the dynamic pressure at the tail different from the free stream dynamic pressure. These are called interference effects and are discussed here before describing the contributions of the tail to C_{mcg} and $C_{m\alpha}$.

2.4.2 Effect of downwash due to wing on angle of attack of tail

Wing is the principal contributor to the lift produced by the airplane. While producing the lift, wing induces an angle of attack on the stream around it. The induced angle is positive ahead of the wing and is called upwash. Behind the wing, the induced angle is negative and is called downwash. It may be recalled that in the lifting line theory, used to calculate flow past a finite wing, a bound vortex is located along the quarter chord line and trailing vortices behind the wing. Using this theory the upwash/downwash distribution can be calculated. Typical distribution is shown in Fig.2.12. Its important features are as follows.

(a) Upwash is zero far ahead of the wing.

(b) Peak in the upwash occurs slightly ahead of the wing.

(c) There is downwash at the wing quarter chord and the downwash angle

$$\varepsilon_{c/4} = (1+ \tau)(C_{Lw} / \pi A_w) \qquad (2.28)$$

where, C_{Lw} is the wing lift coefficient, A_w is the aspect ratio of wing and depends on wing parameters like aspect ratio, taper ratio and sweep.

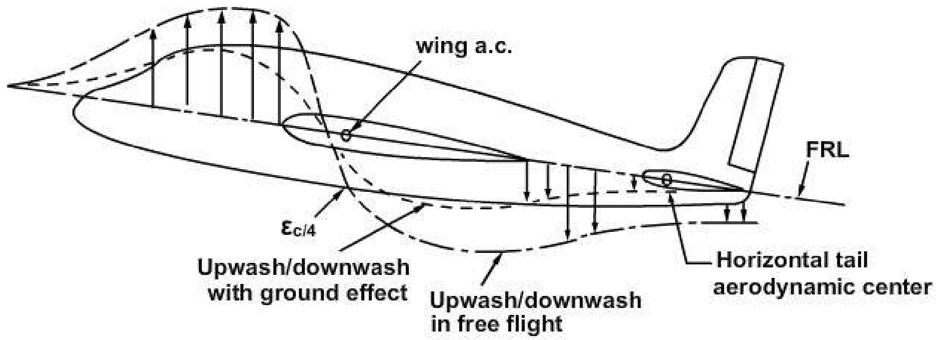


Fig. 2.12 Upwash–downwash field of a wing

(Adapted from Ref.1.12, Chapter 3 with permission from American Institute of Aeronautics and Astronautics, Inc.)

(d) Behind the wing at distances where the tail is located, the downwash angle

(ε_{fb}) is approximately twice of $\varepsilon_{c/4}$.

(e) The upwash/downwash decrease when airplane is near ground as compared to that in free flight.

In a conventional configuration the tail is located behind the wing and would experience downwash i.e. angle of attack of tail would be reduced by ε or

$\alpha_t = \alpha + i_t - \varepsilon$. The value of ε would depend on wing parameters, wing lift coefficient, Mach number, tail parameters and location of the tail with respect to the wing. The common practice is to obtain a value of $d\varepsilon/d\alpha$ in subsonic flow based on

(a) wing aspect ratio, taper ratio and sweep and (b) location of tail aerodynamic centre with respect to wing and subsequently apply corrections for Mach number effect (see Ref.1.12). Appendix „C" explains the procedure to calculate $d\varepsilon/d\alpha$ for a jet airplane.

Remark:

For an elliptic wing the downwash at the aerodynamic centre of the wing ($\varepsilon_{c/4}$)is:

$\varepsilon_{c/4} \approx C_{Lw}/\pi A_w$ (2.29)

Hence, $\varepsilon_{fb} \approx 2\varepsilon_{c/4} = 2\, C_{Lw}/\pi A_w$ (2.30)

Therefore the $(d\varepsilon/d\alpha)_{fb} \approx 2C_{L\alpha w}/\pi A_w$ (2.31)

For example if $A_w = 8$, then $C_{L\alpha w} \approx 2\pi \times 8/(8+2) = 5.08$/radian and

$(d\varepsilon/d\alpha)_{fb} = 2 \times 5.08 / (\pi \times 8) = 0.4$

The tail is generally located far behind the wing and hence the downwash at the tail (ε) is roughly equal to ε_{fb} and Eq.(2.31) can be used to get a rough estimate of $(d\varepsilon/d\alpha)$ even for non-elliptic wings.

2.4.3 Interference effect on dynamic pressure over tail

The dynamic pressure over tail is different from the free stream dynamic pressure due to the following reasons.

(a) Tail may be in the wake of the wing. In a wake the velocity is lowest at the centre and gradually reaches the free stream value (Fig.2.13). The difference between the centre line velocity and the free stream velocity is called velocity defect. The velocity defect depends on the drag coefficient of the wake producing body and the distance behind it. Figure 2.13 shows schematically the wing, the wake centre line and a typical velocity profile of the wake. It is evident that if the tail lies within the wake of the wing, then the dynamic pressure on the tail will be
lower than the free stream dynamic pressure.

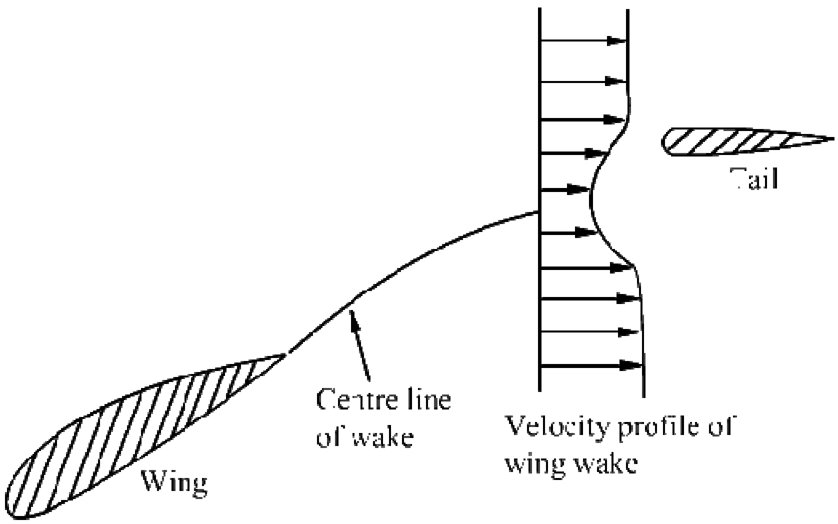


Fig.2.13 Wing, wake centre line and velocity profile of wake – schematic

(b) Some portion of the tail near its root chord is covered by the boundary layer on the fuselage and as such would experience lower dynamic pressure.

(c) In airplanes with engine propeller combination the slip stream of the propeller may pass over the horizontal tail (Fig. 2.14). It may be recalled that the slip stream of a propeller has higher dynamic pressure than that of the free stream. Hence, the propeller slip stream passing over the tail may increase the dynamic pressure over it in comparison to the free stream dynamic pressure.

The ratio of the dynamic pressure experienced by the tail ($\frac{1}{2} \rho V_t^2$) to the free
stream dynamic pressure ($\frac{1}{2} \rho V^2$) is called tail efficiency and denoted by η i.e.

$$\eta = (\tfrac{1}{2} \rho V_t^2) / (\tfrac{1}{2} \rho V^2) \qquad (2.31a)$$

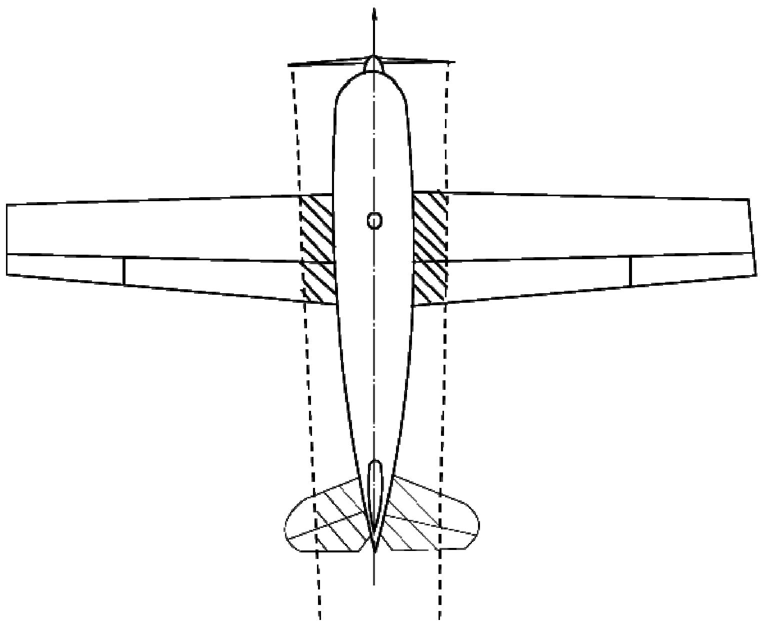


Fig. 2.14 Effect of propeller slip stream on horizontal tail

It is difficult to accurately estimate the value of η. It is assumed between 0.9 and 1.0. It could be more than 1 when the tail is in the slip stream of a propeller.

2.4.4 Expression for C_{mcgt}

With this background the contributions of tail to C_{mcg} and $C_{m\alpha}$ can now be obtained. Figure 2.15 shows schematically, the forces and moment on the tail.

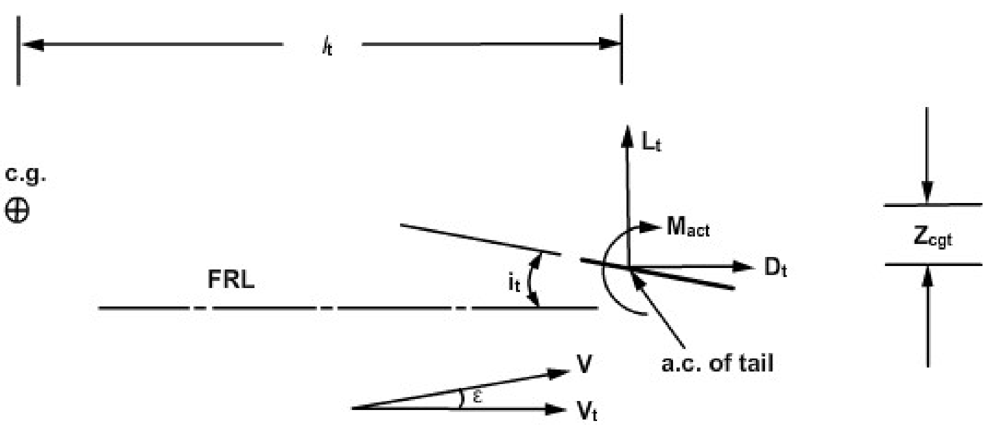


Fig. 2.15 Schematic representation of forces and moment on tail

From Fig.2.15 the angle of attack of the tail is:

$$\alpha_t = \alpha + i_t - \varepsilon = \alpha_w - i_w - \varepsilon + i_t \qquad (2.32)$$

Taking moment about c.g.

$$M_{cgt} = -l_t\,[L_t\cos(\alpha-\varepsilon) - D_t\sin(\alpha-\varepsilon)] + M_{act} - Z_{cgt}[D_t\cos(\alpha-\varepsilon) - L_t\sin(\alpha-\varepsilon)] \tag{2.33}$$

The quantity (α-ε) is generally small and cos (α-ε) is roughly equal to one and ertms involving sin (α-ε) are ignored. M_{act} is also ignored.

Hence,

$$M_{cgt} = -l_t\ L_t \tag{2.34}$$

$$= -l_t\ C_{Lt} \frac{1}{2} \rho V_t^2 S_t \tag{2.35}$$

Consequently,

$$C_{mcgt} = \frac{M_{mcgt}}{\frac{1}{2}\rho V^2 S\bar{c}} = \frac{S_t}{S}\frac{l_t}{\bar{c}}\frac{(1/2)\rho V_t^2}{(1/2)\rho V^2} C_{Lt} \tag{2.36}$$

The term $(S_t/S)(l_t/c)$ is called tail volume ratio and is denoted by V_H. It may be pointed out that the terms $S_t\ l_t$ and $S\bar{c}$ have dimensions of volume. As mentioned earlier, the term $[(1/2)\rho V_t^2]/[(1/2)\rho V^2]$ is called the tail efficiency and denoted by η.

Thus,

$$C_{mcgt} = -V_H \eta C_{Lt};\quad V_H = \frac{S_t}{S}\frac{l_t}{\bar{c}};\quad \eta = \frac{\frac{1}{2}\rho V_t^2}{\frac{1}{2}\rho V^2} \tag{2.37}$$

It may be pointed out that typically, $(S_t/S) \sim 0.2$ to 0.25 and $(l_t/c) \sim 2$ to 3. Hence, V_H lies between 0.4 and 0.7.

2.4.5 Estimation of C_{Lt}

A tail consists of the fixed portion (stabilizer) and the movable portions namely elevator and tab (Fig.2.16a). The tab is located near the trailing edge of the elevator. Its purpose will be explained in chapter 3. The positive deflections of the elevator (δ_e) and of the tab(δ_t)are shown in Fig.2.16b. A positive δ_e produces increase in C_{Lt} and leads to a negative M_{cg}. The changes in lift coefficient of tail due to α_t, δ_e and δ_t are shown in Fig.2.17.

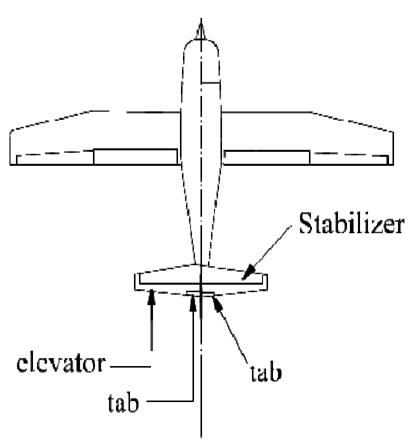


Fig.2.16a Stabilizer, elevator and tab

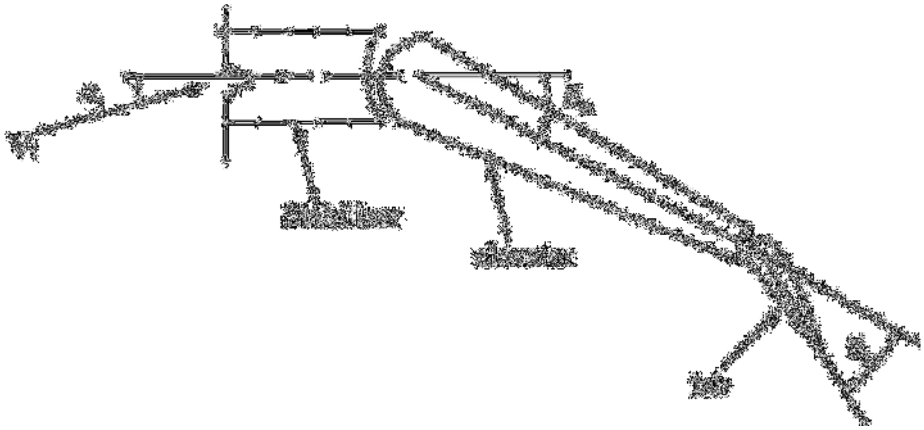


Fig. 2.16b Cross-section of tail with elevator and tab deflected

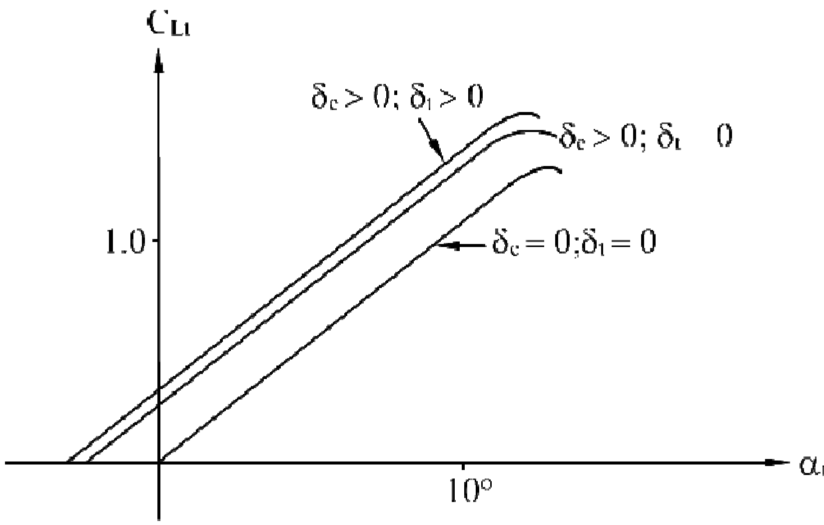


Fig. 2.17 Changes in C_{Lt} due to α_t, δ_e and δ_t

The following may be noted. (a) Generally symmetric airfoils are used on the control surfaces. Hence, C_{Lt} is zero when α_t is zero (b) The elevator is like a flap and a downward (or positive) deflection increases C_{Lt} over and above that due to α_t (Fig.2.17). (c) A positive deflection of tab increases C_{Lt} further (Fig.2.17). (d) Negative deflections of the elevator and tab would have effects opposite of the positive deflections. Taking into account the effects of α_t , δ_e and δ_t, the tail lift coefficient can be expressed as:

$$C_{Lt} = C_{L\alpha t}\,\alpha_t + \frac{\partial C_{Lt}}{\partial \delta_e}\delta_e + \frac{\partial C_{Lt}}{\partial \delta_t}\delta_t \tag{2.38}$$

Note: The expression in Eq.(2.38) is valid only when C_{Lt} vs α curve is linear or the angle of attack is below α_{stall} for the tail.

Now, $\alpha_t =$

$$\alpha - \varepsilon + i_t = \alpha_w - i_w - \varepsilon + i_t \quad (2.39)$$

As noted earlier, ε at a point depends on the wing parameters and location of the point . However, ε is proportional to C_{Lw} i.e. ε = constant x C_{LW}.

Hence,

$$\varepsilon = \frac{d\varepsilon}{dC_{Lw}} C_{Lw} = \frac{d\varepsilon}{d\alpha_w} \frac{d\alpha_w}{dC_{Lw}} C_{Lw} \tag{2.40}$$

Further,

$$\alpha_w = \alpha + i_w \text{ hence, } \frac{d\varepsilon}{d\alpha_w} = \frac{d\varepsilon}{d\alpha}$$

$$\begin{aligned}
\text{Consequently}, \varepsilon &= \frac{d\varepsilon}{d\alpha} \frac{1}{C_{L\alpha w}} C_{Lw} \\
&= \frac{d\varepsilon}{d\alpha} \frac{1}{C_{L\alpha w}} C_{L\alpha w} (\alpha_w - \alpha_{0Lw}) \\
&= \frac{d\varepsilon}{d\alpha} (\alpha + i_w - \alpha_{0Lw}) \\
&= \frac{d\varepsilon}{d\alpha} (i_w - \alpha_{0Lw}) + \frac{d\varepsilon}{d\alpha} \alpha \\
&= \varepsilon_0 + \frac{d\varepsilon}{d\alpha} \alpha; \ \varepsilon_0 = \frac{d\varepsilon}{d\alpha} (i_w - \alpha_{0Lw}).
\end{aligned} \tag{2.41}$$

Remark:

Reference 1.1 uses the following approximate expression:

(2.43)

$$\varepsilon = \frac{2}{\pi A_w} C_{L\alpha w}(\alpha + i_w - \alpha_{0Lw})$$

$$\varepsilon = \frac{2}{\pi A_w} C_{L\alpha w}(i_w - \alpha_{0Lw}) + \frac{2}{\pi A_w} C_{L\alpha w}\alpha$$

$$\varepsilon = \varepsilon_0 + \frac{2}{\pi A_w} C_{L\alpha w}\alpha$$

$$\text{Hence, } \frac{d\varepsilon}{d\alpha} \approx \frac{2C_{L\alpha w}}{\pi A_w}$$

(2.43)
which is the downwash far behind an elliptic wing. Substituting this, Eq.(2.42) gives:

As mentioned earlier, dε/dα depends on the wing parameters, location of the tail relative to wing and the Mach number.

Noting that α_T $\alpha_w = \frac{C_{Lw}}{C_{L\alpha w}} + \alpha_{0Lw}$,

$$\alpha_t = \frac{C_{Lw}}{C_{L\alpha w}} + \alpha_{0Lw} - i_w - \frac{d\varepsilon}{d\alpha}\frac{C_{Lw}}{C_{L\alpha w}} + i_t$$

$$= \alpha_{0Lw} - i_w + i_t + \frac{C_{Lw}}{C_{L\alpha w}}(1 - \frac{d\varepsilon}{d\alpha}) \qquad (2.44)$$

Alternatively, we can write

$$\alpha_t = \alpha - \varepsilon_0 - \frac{d\varepsilon}{d\alpha}\alpha + i_t \qquad (2.45)$$

$$= i_t - \varepsilon_0 + \alpha(1 - \frac{d\varepsilon}{d\alpha})$$

Putting these together, yields:

$$C_{Lt} = C_{L\alpha t}[i_t - \varepsilon_0 + \alpha(1 - \frac{d\varepsilon}{d\alpha})] + \frac{\partial C_{Lt}}{\partial \delta_e}\delta_e + \frac{\partial C_{Lt}}{\partial \delta_t}\delta_t \qquad (2.46)$$

2.4.6 Revised expression for C_{mcgt}

Substituting for C_{Lt} in Eq.(2.37) gives:

$$C_{mcgt} = -V_H \eta\, C_{L\alpha t}[i_t - \varepsilon_0 + \alpha(1 - \frac{d\varepsilon}{d\alpha}) + \tau\ \delta_e + \tau_{tab}\delta_{tab}]$$

$$= -C_{mot} - V_H\ \eta\ C_{L\alpha t}\ [\alpha(1 - \frac{d\varepsilon}{d\alpha}) + \tau\ \delta_e + \tau_{tab}\ \delta_{tab}] \qquad (2.47)$$

$$\text{where, } \tau = \frac{\partial C_{Lt}}{\partial \delta_e} / \frac{\partial C_{Lt}}{\partial \alpha};\ \ \tau_{tab} = \frac{\partial C_{Lt}}{\partial \delta_t} / \frac{\partial C_{Lt}}{\partial \alpha} \qquad (2.48)$$

$$\text{and } C_{m\alpha t} = -V_H \eta C_{L\alpha t}(i_t - \varepsilon_0) \tag{2.49}$$

2.4.7 $C_{m\alpha t}$ in stick-fixed case

It may be pointed out that the pilot moves the elevator through the forward and backward movements of the control stick. Further, depending on the values of chosen flight speed and altitude, the pilot adjusts the positions of the elevator to make C_{mcg} equal to zero. In small airplanes, like the general aviation airplanes, the pilot continues to hold the stick and maintain the elevator deflection. In this background the analysis of the static stability of an airplane where the control deflection remains same even after disturbance is called static stability stick-fixed.

Hence, to obtain an expression for $C_{m\alpha t}$ in stick-fixed case it is assumed that the elevator deflection (δ_e) and the tab deflection (δ_t) remain unchanged after the disturbance. Accordingly when Eq.(2.47) is differentiated with respect to α ,the derivatives of δ_e and δ_t are zero i.e., in this case, $(d\delta_e /d\alpha) = (d\delta_t /d\alpha) = 0$ and the following result is obtained:

$$(C_{m\alpha t})_{\text{stick fixed}} = -V_H \eta C_{L\alpha t}\left(1 - \frac{d\varepsilon}{d\alpha}\right) \tag{2.50}$$

Remarks:

i) $C_{m\alpha t}$ is negative. To illustrate this, consider typical values as: $\eta = 0.9$, $V_H = 0.5$, $C_{L\alpha t} = 4.0$ per radian and $d\varepsilon/d\alpha = 0.4$. Then,

$C_{m\alpha t} = -0.5 \times 0.9 \times 4 \times (1-0.4) = -1.08$/radian.

ii) V_H depends on (S_t/S) and $(l_t/\bar{c})$. Hence, the contribution of tail to stability ($C_{m\alpha t}$) can be increased in magnitude by increasing (S_t/S) or $(l_t/\bar{c})$ i.e. by increasing the area of the horizontal tail or by shifting the tail backwards.

iii) C_{m0} is the value of C_{mcg} when α is zero. It (C_{m0}) is the sum of terms like C_{mow},C_{mot} etc. This value (C_{m0}) can be adjusted by changing C_{m0t}. In this context we observe from Eq.(2.49) that: $C_{m0t} = -V_H \eta C_{L\alpha t}(i_t - \varepsilon_0)$

This suggests that by choosing a suitable value of i_t , the value of C_{mo} can be adjusted. This would permit trim, with zero elevator deflection, at a chosen value of lift coefficient (see Fig.2.6 and example 2.5). The chosen value of C_L for this purpose is invariably the value of C_L during cruise. This serves as criterion for selecting tail setting.

iv) In the beginning of this section a reason for examination of the stick-fixed stability was given by considering the case of general aviation airplane. However, the analysis of stick-fixed stability is carried out for all airplanes and the level of $(C_{m\alpha t})_{\text{stick-fixed}}$ decides the elevator deflection required in steady flight and in manoeuvres (see subsections 2.12.3 and 4.2).

2.5 Contributions of fuselage to C_{mcg} and $C_{m\alpha}$

Fuselage and nacelle are classified as bodies. The steps for estimating the contributions of a body to C_{mcg} and $C_{m\alpha}$ are based on the descriptions in chapter 2 , of Ref.1.1, chapter 5 of Ref.1.7 and chapter 3 of Ref.1.12. In this

approach, the contribution of the body to $C_{m\alpha}$ is estimated based on the slender body theory and subsequently applying corrections for the effects of (a) finite fineness ratio, (b) non-circular cross section, (c) fuselage camber and (d) downwash due to wing.

2.5.1 Contribution of body to $C_{m\alpha}$ based on slender body theory

The potential flow past and an axisymmetric slender body was studied by Munk in 1924 (see Ref.1.1, chapter 2 for bibliographic details). He showed that a body at an angle of attack has a pressure distribution as shown in Fig.2.18 and produces no net force, but a moment. He showed that the rate of change of moment with angle of attack α, in radians, is given by:

$$\frac{dM}{d\alpha} = 2q \times \text{volume of body}\,;\, q = \frac{1}{2}\rho V^2$$

Alternatively when α is in degrees,

$$\frac{dM}{d\alpha} = \frac{\text{Volume of body} \times q}{28.7}$$

(2.

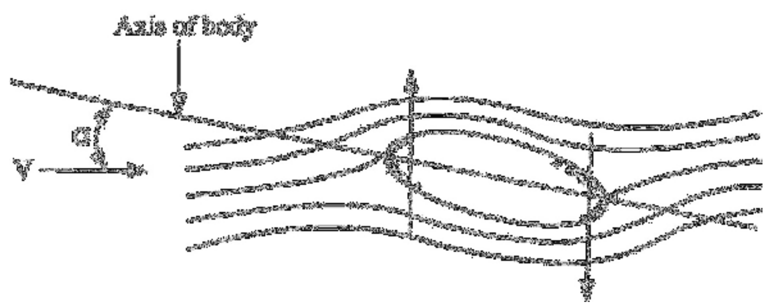


Fig.2.18 Streamlines and potential flow pressure distribution on an axisymmetric body; the negative and positive signs indicate respectively that the local pressure is lower or higher than the free stream pressure

Remark:

In a viscous flow, the pressure distribution about the body changes and it experiences lift and drag.

2.5.2 Correction to moment contribution of fuselage for fineness ratio

Generally the fuselage has a finite length. For such a fuselage Multhopp in

1942 (see Ref.1.1, chapter 2 for bibliographic details) suggested the following correction to Eq.(2.51).

$$\frac{dM}{d\alpha} = \frac{(k_2-k_1)}{28.7} q \times \text{volume of body} \qquad 2.52$$

Where, (k_2-k_1) is a factor which depends on the fineness ratio (l_f/d_e) of the body;

l_f is the length of the body and d_e is the equivalent diameter defined as:

$(\pi/4)d_e^2 \stackrel{2}{=}$ max. cross sectional area of fuselage.

Figure 2.19 presents variation of (k_2-k_1) with fineness ratio

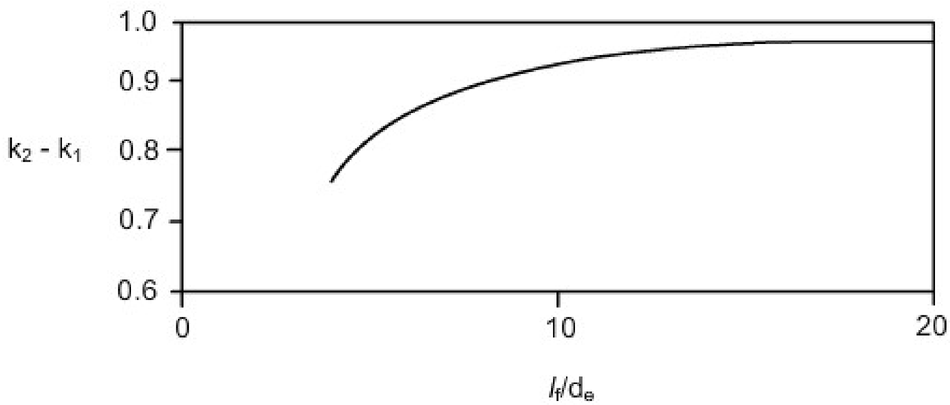


Fig.2.19 Correction to moment contribution of fuselage for fineness ratio

(Adapted from Ref.2.2, section 4.2.1.1)

2.5.3 Correction to moment contribution of fuselage for non-circular cross section

For a fuselage of non-circular cross-section, Eq.(2.52) is modified as:

$$\frac{dM}{d\alpha} = \frac{(k_2-k_1)q}{28.7}\int_0^{l_f}\frac{\pi}{4}w_f^2\,dx = \frac{(k_2-k_1)q}{36.5}\int_0^{l_f}w_f^2\,dx \qquad (2.53)$$

where, w_f is the local width of the fuselage.

Hence, the contribution of fuselage to $C_{m\alpha}$ can be expressed as :

$$C_{m\alpha f} = \frac{\frac{dM}{d\alpha}}{\frac{1}{2}\rho V^2 S\bar{c}} \qquad (2.54)$$

2.5.4 Correction to moment contribution due to fuselage for fuselage camber and downwash due to wing

In an airplane, the flow past a fuselage is affected by the upwash-downwash field of the wing (Fig.2.12). Further, the midpoints of the fuselage cross sections may not lie in a straight line. In such a case the fuselage is said to have a camber (Fig.2.20). A fuselage with camber would produce a pitching moment coefficient (C_{mof}) even when FRL is at zero angle of attack.

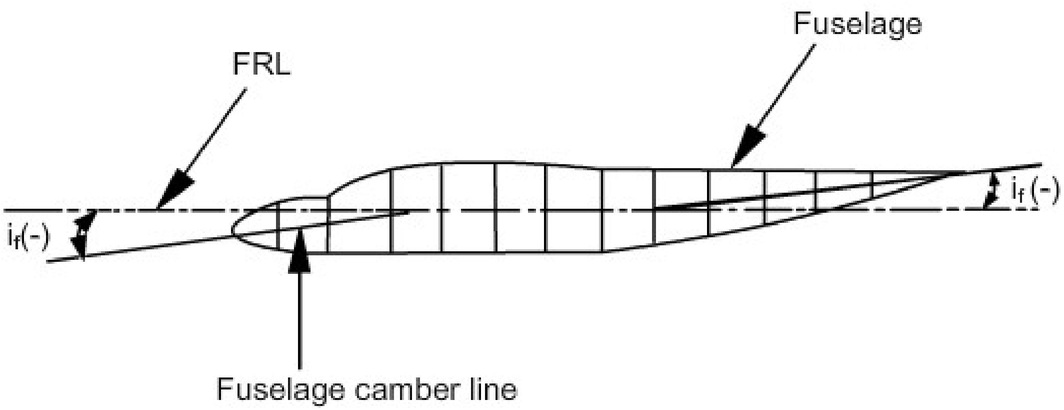


Fig.2.20 Fuselage with camber

For a fuselage with camber, C_{mcgf} is expressed as:

$$C_{mcgf} = C_{m0f} + C_{m\alpha f}\,\alpha \qquad (2.55)$$

with

$$C_{m0f} = \frac{k_2 - k_1}{36.5 S\bar{c}} \sum_{x=0}^{l_f} w_f^2 (\alpha_{0Lf} + i_f)\Delta x \quad \text{and} \qquad (2.56)$$

$$C_{m\alpha f} = \frac{k_2 - k_1}{36.5 S\bar{c}} \sum_{x=0}^{l_f} w_f^2 \frac{d\varepsilon}{d\alpha}\Delta x \qquad (2.57)$$

where, (a) w_f is the average width over a length Δx of fuselage (Fig.2.21) (b) i_f is the incidence angle of fuselage camber line with respect to FRL. It is taken negative when there is nosedrop or aft upsweep (Fig.2.20). (c) α_{0Lf} is the zero lift angle of wing relative to FRL i.e. $\alpha_{0Lf} = \alpha_{0Lw} + i_w$ and (d) $d\varepsilon/d\alpha$ is the derivative with α of the local value of upwash / downwash along the fuselage.

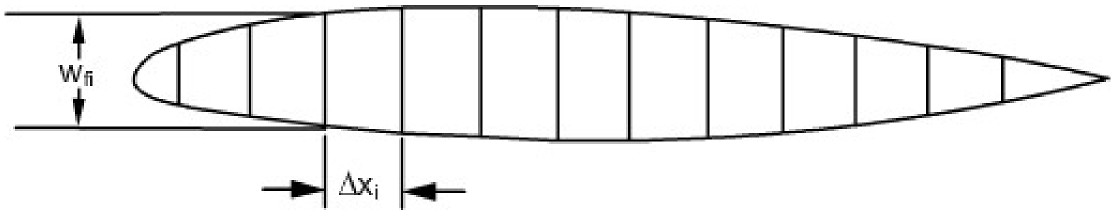


Fig.2.21 Division of fuselage for calculation of C_{m0f}

Though $d\varepsilon/d\alpha$ along the fuselage can be calculated from an approach like the lifting line theory, the following emprical procedure is generally regarded adequate for evaluating $C_{m\alpha f}$.

a) The fuselage is divided into segments as shown in Fig.(2.22).

b) The local value of, $d\varepsilon/d\alpha$ ahead of the wing is denoted by $d\varepsilon_u /d\alpha$. It is estimated from Fig.(2.23). For the segment immediately ahead of the wing (section 5 in Fig.2.22) the value of $d\varepsilon_u /d\alpha$ varies rapidly and is estimated from the curve 'b' in Fig.2.23 (see example 2.4). For other segments ahead of wing, the curve 'a' in the same figure is used to estimate $d\varepsilon_u /d\alpha$.

(c) For the portion of the fuselage covered by the wing root (length 'c' indicated in Fig.2.22) $d\varepsilon/d\alpha$ is taken as zero. Actually, the contribution of this portion is taken to be zero as, this portion is accounted for under the wing area.

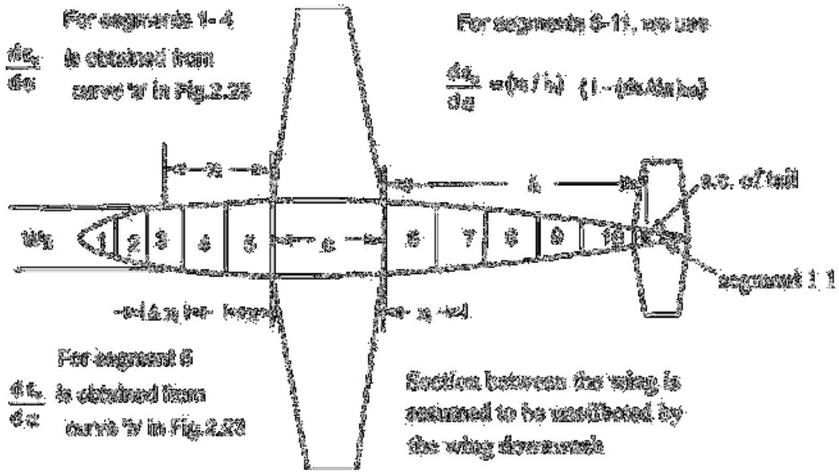


Fig.2.22 Division of fuselage for calculation of $C_{m\alpha f}$

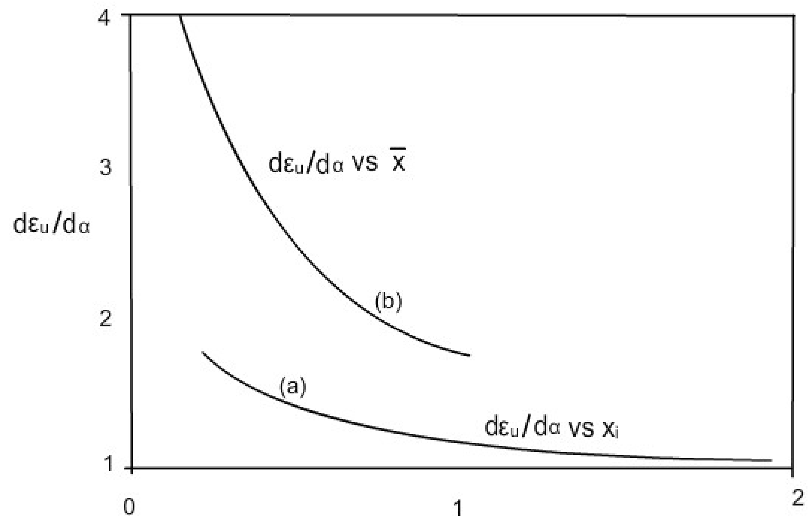


Fig.2.23 Upwash field ahead of wing

(Adapted from Ref.2.2, section 4.2.2.1)

(d) For the portion of the fuselage behind the wing (segments 6 to 11 in Fig.2.22), $d\varepsilon/d\alpha$ is assumed to vary linearly from 0 to $\{1-(d\varepsilon/d\alpha)_{tail}\}$ where $(d\varepsilon/d\alpha)_{tail}$ is the value of $(d\varepsilon/d\alpha)$ at a.c. of tail.

Hence, $$\frac{d\varepsilon}{d\alpha} = \frac{x_i}{l_t}\left[1-\left(\frac{d\varepsilon}{d\alpha}\right)_{tail}\right] \qquad (2.58)$$

The procedure is illustrated in example 2.4 for a low speed airplane and in Appendix ‘C’ for a jet airplane

Remarks:

i) In Ref.2.2, the quantity $d\varepsilon/d\alpha$ of Eq.(2.57) is written as $(1+ d\varepsilon/d\alpha)$ and values of $d\varepsilon/d\alpha$ therein are accordingly lower by one as compared to those in Fig. 2.23 ii) The values in Fig.2.23 are for a $C_{L\alpha WB}$ of 0.0785/deg. $C_{L\alpha WB}$ is the slope of lift curve of the wing-body combination which is roughly equal to $C_{L\alpha W}$ when the aspect ratio of the wing is greater than five. For other values of $C_{L\alpha W}$ multiply the values of $d\varepsilon/d\alpha$ by a factor of $(C_{L\alpha W}/0.0785)$. Note that $C_{L\alpha W}$ is in deg^{-1}. See also example 2.4.

2.5.5 Contribution of nacelle to $C_{m\alpha}$

The contribution of nacelle to $C_{m\alpha}$ can be calculated in a manner similar to that for the fuselage. Generally it is neglected.

Chapter 2

Lecture 8

Longitudinal stick–fixed static stability and control – 5

Topics

2.6 Contributions of power plant to C_{mcg} and $C_{m\alpha}$

2.6.1 Direct contributions of powerplant to C_{mcg} and $C_{m\alpha}$

2.6.2 Indirect contributions of powerplant to C_{mcg} and $C_{m\alpha}$

2.7 General remarks – slope of lift curve ($C_{L\alpha}$) and angle of zero lift (α_{0L}) of airplane

2.7.1 Slope of lift curve ($C_{L\alpha}$) of the airplane

2.7.2 Angle of zero lift of the airplane

2.8 C_{mcg} and $C_{m\alpha}$ of entire airplane

2.9 Stick-fixed neutral point

2.9.1 Neutral point power-on and power-off

2.10 Static margin

2.11 Neutral point as aerodynamic centre of entire airplane

2.6 Contributions of power plant to C_{mcg} and $C_{m\alpha}$

The contributions of power plant to C_{mcg} and $C_{m\alpha}$ have two aspects namely direct contribution and indirect contribution.

2.6.1 Direct contribution of power plant to C_{mcg} and $C_{m\alpha}$

The direct contribution appears when the direction of the thrust vector does not coincide with the line passing through the c.g.(Fig.2.24). The direct contribution is written as :

$$M_{cgp} = T \times Z_p \qquad (2.59)$$

where, T is the thrust and Zp is the perpendicular distance of thrust line from FRL; positive when c.g. is above thrust line.

In non-dimensional form Eq.(2.59) is expressed as:

$$C_{mcgp} = M_{cgp}/(\tfrac{1}{2}\,\rho V^2 S\, c) \tag{2.60}$$

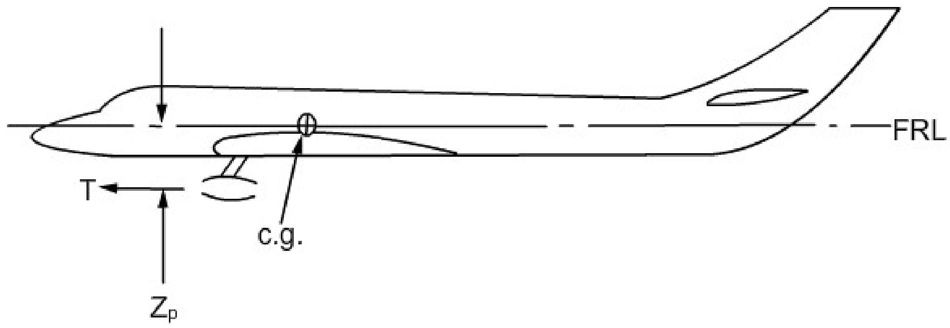


Fig.2.24 Contribution of thrust to C_{mcg}

The thrust required varies with flight speed and altitude. Hence, C_{mcgp} would vary with flight condition. However, the thrust setting does not change during the disturbance and hence, there is no contribution to $C_{m\alpha}$. This fact is also mentioned in Ref.1.9. p.506.

The contribution to $C_{m\alpha}$ comes from another cause. Consider a propeller at an angle of attack as shown in Fig.2.25. The free stream velocity (V) is at an angle (α)to the propeller axis. As the air stream passes through the propeller it leaves in a nearly axial direction. This change of direction results in a normal force (N_p) in addition to the thrust (T).

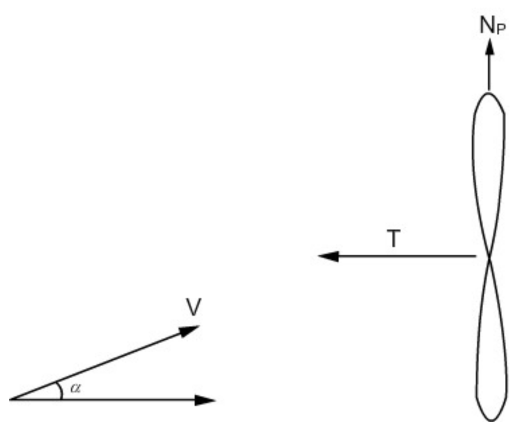


Fig.2.25 Propeller at angle of attack

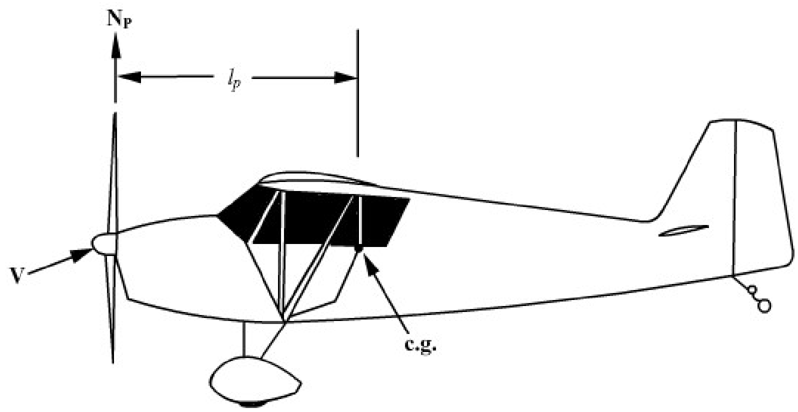


Fig.2.26 Contribution to $C_{m\alpha}$ from normal force due to propeller

N_p acts at distance l_p from the c.g. (Fig.2.26) and hence, produces a moment N_p x l_p. The value of N_p depends on the angle of attack of the propeller and hence the term N_p x l_p depends on α. This will contribute to $C_{m\alpha}$. $C_{m\alpha}$ due to normal force depends on many factors like thrust setting, number of blades in the propeller and advance ratio.

Remarks:

i) It is evident from Fig.2.26 that when the propeller is ahead of c.g., the contribution to $C_{m\alpha}$ due to normal force would be positive or destabilizing. In a pusher airplane, where the propeller is near the rear end of the airplane, the contribution of normal force to $C_{m\alpha}$ will be negative and hence stabilizing.

ii) In the case of a jet engine at an angle of attack, the air stream enters the intake at that angle and its direction has to change as the stream passes through the engine. This change of direction will also produce a normal force N_p and consequently contribute to $C_{m\alpha}$.

2.6.2 Indirect contributions of power plant to C_{mcg} and $C_{m\alpha}$

The effect of propeller on the horizontal tail has been discussed in section 2.4.3. In the case of an airplane with a jet engine, the exhaust expands in size as it moves downwards and entrains the surrounding air. This would induce an angle to the flow; the induced angle would be positive in the region below the jet and negative in the region above the jet. In military airplanes where the engine is located in rear fuselage the engine exhaust would affect the horizontal tail, generally located above the rear fuselage, by inducing a downwash in addition to that due to wing. This effect will also come into picture in case of passenger airplanes with rear mounted engines. To alleviate this, the horizontal tail is mounted above the vertical tail (see configurations of Boeing MD-87 and Gulf stream V in Ref.2.3).

Remarks:

i) The contribution of engine depends also on the engine power setting which in turn depends on flight condition or C_L. Hence, the level of stability ($C_{m\alpha}$) will depend on C_L and also will be different when engine is off or on.

ii) It is difficult to accurately estimate the effects of power on $C_{m\alpha}$. A rough estimate would be (Ref.1.7, chapter 5) :

$(dC_m / dC_L)_p = 0.04$ or $C_{m\alpha p} = 0.04 C_{L\alpha}$ (2.60a)

2.7 General Remarks:

2.7.1 Slope of lift curve ($C_{L\alpha}$) and angle of zero lift (α_{0L}) of the airplane:

Let, L denote lift of airplane. Then, $L = L_{wb} + L_t$.

For airplanes with large aspect ratio wings (A>5), the lift of the wing body combination is approximately equal to lift produced by the gross wing i.e.. $L_{wb} \approx L_w$ Noting that $L_t = \frac{1}{2}\rho V_t^2 S_t C_{L\alpha t} (\alpha - \varepsilon + i_t)$ and $L_w = \frac{1}{2}\rho V^2 S C_{Lw}$; the slope of the lift curve of the airplane ($C_{L\alpha}$) can be written as :

$C_{L\alpha} = C_{L\alpha w} + \eta (S_t/S) C_{L\alpha t} \{1 - (d\varepsilon/d\alpha)\}$ (2.60b)

Reference 1.8 b gives expressions for corrections to obtain $C_{L\alpha wb}$ from $C_{L\alpha w}$ (see also Appendix C section 5).

2.7.2 Angle of zero lift (α_{0L}) for airplane:

Assuming that the wing is set such that during cruise the angle of attack of the airplane (α_{cr}) is zero, the lift coefficient during cruise (C_{Lcr}) can be written as :

$C_{Lcr} = C_{L\alpha} (\alpha_{cr} - \alpha_{0L}) = C_{L\alpha} (0 - \alpha_{0L})$

Hence, $\alpha_{0L} = - C_{Lcr} / C_{L\alpha}$ (2.60c)

2.8 C_{mcg} and $C_{m\alpha}$ of entire airplane

The important result of the last few sections can be recapitulated as follows.

$$C_{mcg} = (C_{mcg})_w + (C_{mcg})_f + (C_{mcg})_n + (C_{mcg})_p + (C_{mcg})_{ht} \quad (2.12)$$

$$C_{m\alpha} = (C_{m\alpha})_w + (C_{m\alpha})_f + (C_{m\alpha})_n + (C_{m\alpha})_p + (C_{m\alpha})_{ht} \quad (2.13)$$

The wing Contribution is:

$$C_{mcgw} = C_{macw} + C_{Lw}\left(\frac{x_{cg}}{c} - \frac{x_{ac}}{c}\right) \quad (2.17)$$

$$\begin{aligned} C_{Lw} &= C_{L\alpha w}(\alpha + i_w - \alpha_{0Lw}) \\ &= C_{L0w} + C_{L\alpha w}\,\alpha \; ; \; C_{L0w} = C_{L\alpha w}(i_w - \alpha_{0Lw}) \end{aligned} \quad (2.18)$$

$$C_{mcgw} = C_{macw} + C_{L0w}\left(\frac{x_{cg}}{c} - \frac{x_{ac}}{c}\right) + C_{L\alpha w}\,\alpha\left(\frac{x_{cg}}{c} - \frac{x_{ac}}{c}\right) \quad (2.19)$$

$$C_{m0w} = C_{macw} + C_{L0w}\left(\frac{x_{cg}}{c} - \frac{x_{ac}}{c}\right) \quad (2.19a)$$

$$(C_{m\alpha})_w = C_{L\alpha w}\left(\frac{x_{cg}}{c} - \frac{x_{ac}}{c}\right) \quad (2.20)$$

The tail Contribution is:

$$C_{mcgt} = -V_H\, \eta\, C_{Lt} \quad (2.37)$$

$$C_{Lt} = C_{L\alpha t}\,\alpha_t + C_{L\delta e}\,\delta_e + C_{L\delta t}\,\delta_t \quad (2.38)$$

$$\alpha_t = \alpha - \varepsilon + i_t = \alpha_w - i_w - \varepsilon + i_t \quad (2.39)$$

$$\varepsilon = \varepsilon_0 + \frac{d\varepsilon}{d\alpha}\alpha ; \; \varepsilon_0 = \frac{d\varepsilon}{d\alpha}(i_w - \alpha_{0Lw}) \quad (2.41)$$

$$C_{Lt} = C_{L\alpha t}\left\{i_t - \varepsilon_0 + \alpha\left(1 - \frac{d\varepsilon}{d\alpha}\right)\right\} + C_{L\delta e}\,\delta_e + C_{L\delta t}\,\delta_t \quad (2.46)$$

$$C_{mcgt} = -V_H\, \eta\, C_{L\alpha t}\, \{i_t - \varepsilon_0 + \alpha\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \tau\,\delta_e + \tau_{tab}\delta_t\} \quad (2.47)$$

$$\tau = \frac{C_{L\delta e}}{C_{L\alpha t}} \; ; \; \tau_{tab} = \frac{C_{L_{\delta_t}}}{C_{L_{\alpha_t}}}$$

$$(C_{m\alpha t})_{stick\text{-}fixed} = -V_H\, \eta\, C_{L\alpha t}\left(1 - \frac{d\varepsilon}{d\alpha}\right) \quad (2.50)$$

The contributions of fuselage, nacelle and power are expressed together as:

$$(C_m)_{f,n,p} = (C_{m0})_{f,n,p} + (C_{m\alpha})_{f,n,p}\ \alpha \tag{2.61}$$

Substituting various expressions in Eqs.(2.12) and (2.13) gives:

$$C_{mcg} = C_{m0} + C_{m\alpha}\,\alpha + C_{m\delta e}\,\delta_e \tag{2.62}$$

$$C_{m0} = C_{macw} + C_{L0W}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) + (C_{m0})_{f,n,p} - V_H\ \eta\ C_{L\alpha t}\{i_t - \varepsilon_0 + \tau_{tab}\,\delta_t\} \tag{2.63}$$

$$C_{m\delta e} = -V_H\,\eta\,C_{L\alpha t}\,\tau \tag{2.64}$$

$$(C_{m\alpha})_{stick\ fixed} = C_{L\alpha w}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) + (C_{m\alpha})_{f,n,p} - V_H\,\eta\,C_{L\alpha t}\left(1 - \frac{d\varepsilon}{d\alpha}\right) \tag{2.65}$$

Typical contributions of the individual components and their sum, namely C_{mcg} for a low subsonic airplane are shown in Fig.2.27. The details of the calculations are given in example 2.4.

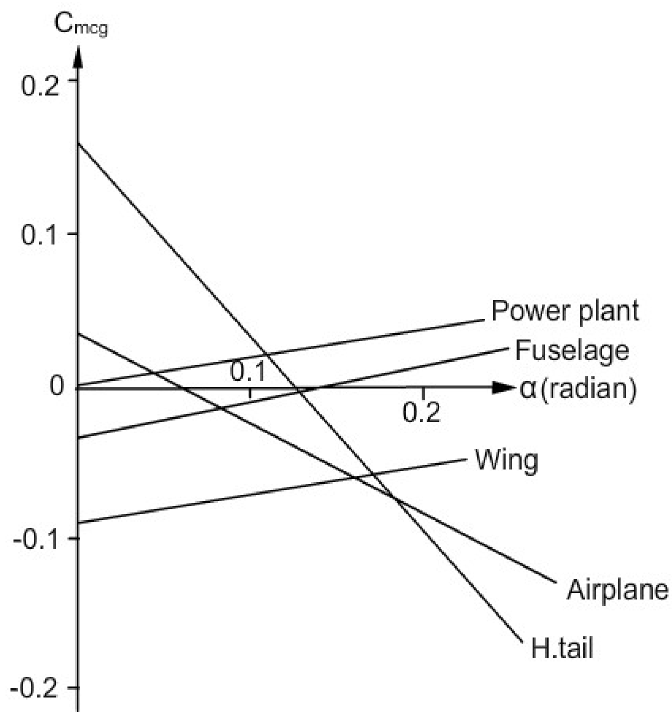


Fig.2.27 C_{mcg} vs α for a low subsonic airplane

Following observations can be made in this case.

(a)C_{mow} has an appreciable negative value.

(b)$C_{m\alpha w}$ depends on the product of $C_{L\alpha w}$ and $\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right)$. In the case considered n example 2.4, the c.g. is at $0.295\bar{c}$ and the a.c. is at $0.25\bar{c}$. Since, c.g. is aft of the aerodynamic centre, the contribution of wing is destabilizing (Fig.2.27).

(c) C_{mof} has small negative value and $C_{m\alpha f}$ has small positive value, indicating a slight destabilizing contribution from fuselage (Fig.2.27).

(d) C_{mot} is positive and $C_{m\alpha t}$ has a large negative value (Fig.2.27).

(e) The line corresponding to the sum of all the contribution (wing+ fuselage+ power+tail) is the C_{mcg} vs α curve for the whole airplane. The contribution of nacelle is ignored. It is seen that the large negative contribution of tail renders $C_{m\alpha}$ negative and the airplane is stable.

2.9 Stick-fixed neutral point

It may be pointed out that the c.g. of the airplane moves during flight due to consumption of fuel. Further, the contribution of wing to $C_{m\alpha}$ depends sensitively on the location of the c.g. as it is proportional to $\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right)$. When the c.g. moves aft, x_{cg} increases and the wing contribution becomes more and more positive. There is a c.g. location at which $(C_{m\alpha})_{stick\text{-}fixed}$ becomes zero. This location of c.g. is called the stick-fixed neutral point. In this case, the airplane is neutrally stable. Following Ref.1.1 this location of the c.g. is denoted as x_{NP}. If the c.g. moves further aft, the airplane will become unstable. The C_m vs. α curves for the statically stable, neutrally stable and unstable cases are schematically shown in Fig.2.28.

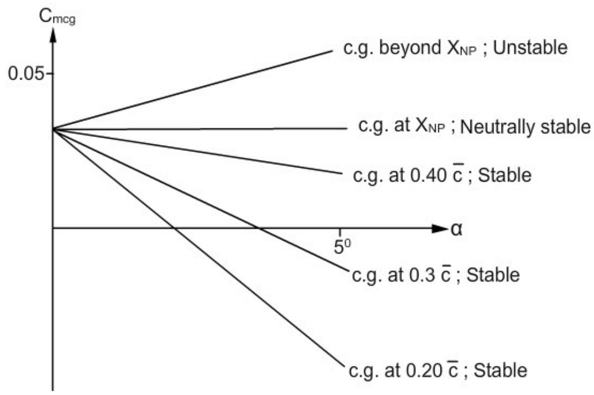


Fig.2.28 Changes in static stability with movement of c.g. (Schematic)

An expression for x_{NP} can be obtained by putting $C_{m\alpha} = 0$ and $x_{cg} = x_{NP}$, in Eq.(2.65) i.e.

mα f,n,p H Lαt dα

$$0 = C_{L\alpha w}\left(\frac{x_{NP}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) + (C_{m\alpha})_{f,n,p} - V_H \eta C_{L\alpha t}\left(1-\frac{d\varepsilon}{d\alpha}\right) \quad (2.66)$$

$$\text{Hence, } \frac{x_{NP}}{\bar{c}} = \frac{x_{ac}}{\bar{c}} - \frac{1}{C_{L\alpha w}}\left\{(C_{m\alpha})_{f,n,p} - V_H \eta C_{L\alpha t}\left(1-\frac{d\varepsilon}{d\alpha}\right)\right\} \quad (2.67)$$

–

Example 2.4 illustrates the steps involved in arriving at the neutral point.

2.9.1 Neutral point power-on and power-off

The contribution of power is generally destabilizing and hence, the airplane will be more stable when engine is off. In other words, x_{NP} power off is behind x_{NP} power on.

2.10 Static margin

Noting the definition of $\frac{x_{NP}}{\bar{c}}$ from Eq.(2.67), the Eq.(2.65) can be rewritten as :

$$(C_{m\alpha})_{\text{stick-fixed}} = C_{L\alpha w}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{NP}}{\bar{c}}\right) \quad (2.68)$$

Thus, $(C_{m\alpha})_{stick\text{-}fixed}$ is proportional to $\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{NP}}{\bar{c}}\right)$ and a term called static margin is defined as:

$$\text{Static margin} = \left(\frac{x_{NP}}{\bar{c}} - \frac{x_{cg}}{\bar{c}}\right) \tag{2.69}$$

$$\text{Consequently, } (C_{m\alpha})_{stick\text{-}fixed} = -C_{L\alpha w} \times (\text{static margin}) \tag{2.70}$$

$$\text{and} \quad \left(\frac{dC_m}{dC_L}\right)_{stick\text{-}fixed} = -(\text{static margin})$$

$$= \frac{1}{C_{L\alpha}}(C_{m\alpha})_{stick\text{-}fixed} \tag{2.71}$$

It may be noted that static margin, by definition, is positive for a stable airplane.

2.11 Neutral point as the aerodynamic centre of entire airplane

To explain the above concept, the derivation of the expression for neutral point in the Ref.1.10. chapter 2 is briefly described. The wing contribution (C_{mcgw}) is expressed as:

$$C_{mcgw} = C_{macw} + \alpha_w \; C_{L\alpha w} \left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right)$$

The contributions of fuselage and nacelle are accounted for by treating them as changes in the following quantities: (a) pitching moment coefficient is changed from C_{macw} to C_{macwb}, (b) the angle of attack is changed from α_w to α_{wb}, (c) the slope of the lift curve is changed from $C_{L\alpha w}$ to $C_{L\alpha wb}$ and (d) aerodynamic centre is change from x_{ac} to x_{acwb}. The suffix 'wb' indicates combined effects of wing body and nacelle. Consequently,

$$C_{mcgwb} = C_{macwb} + \alpha_{wb} \; C_{L\alpha wb} \left(\frac{x_{cg}}{\bar{c}} - \frac{x_{acwb}}{\bar{c}}\right)$$

The contribution of power is expressed as C_{mcgp}.

The contribution of the horizontal tail is expressed as :

$$C_{mcgt} = -\bar{V}_H \, C_{Lt} \text{; note } \eta = 1.0 \text{ (assumed)}$$

$$\text{Where, } \bar{V}_H = \frac{S_t}{S}\frac{\bar{l}_t}{\bar{c}}$$

$\bar{l}_t$ = distance between the aerodynamic centre of the wing-body-nacelle combination (x_{acwb}) and the aerodynamic centre of the horizontal tail.

It is assumed that the C_L and $C_{L\alpha}$ of the airplane are approximately equal to C_{Lwb} and $C_{L\alpha wb}$ respectively. The expression for C_{mcg} can now be written as:

$$C_{mcg} = C_{macwb} + C_L \left(\frac{x_{cg}}{\bar{c}} - \frac{x_{acwb}}{\bar{c}}\right) - \bar{V}_H \, C_{Lt} + C_{mcgp} \qquad (2.62a)$$

$$\text{or } C_{m\alpha} = C_{L\alpha} \left(\frac{x_{cg}}{\bar{c}} - \frac{x_{acwb}}{\bar{c}}\right) - \bar{V}_H \, C_{L\alpha t} + C_{m\alpha p} \qquad (2.65a)$$

The neutral point, x_{NP} , is given by:

$$\frac{x_{NP}}{\bar{c}} = \frac{x_{acwb}}{\bar{c}} - \frac{1}{C_{L\alpha}}(C_{map} - \bar{V}_H C_{L\alpha t}) \qquad (2.67a)$$

$$\text{or } C_{m\alpha} = C_{L\alpha}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{NP}}{\bar{c}}\right) = -C_{L\alpha} \times (\text{static margin}) \qquad (2.70a)$$

It may be recalled that the aerodynamic centre of an aerofoil is the point about which the pitching moment is constant with angle of attack. Similarly, the aerodynamic centre of the wing (x_{ac}), by definition, is the point about which C_{macw} is constant with angle of attack. With this background, the quantity x_{acwb} can be called as the aerodynamic centre of the wing - body - nacelle combination. Further, when the c.g. is at neutral point, $C_{m\alpha}$ is zero or C_{mg} is constant with α. This may be the reason Ref.1.10, chapter 2 refers the neutral point as the aerodynamic centre of the entire airplane.

Remark:

There are some differences in the expressions on the right hand sides of Eq.(2.67) and (2.67a) and Eq.(2.70) and (2.70a). These differences are due to slight difference in treatment of the contributions of individual components. The differences in Eq.(2.70) and (2.70a) can be reconciled by noting that for airplanes with large aspect ratio wings , $C_{L\alpha} \approx C_{L\alpha w}$. Reference 1.12, chapter 3 also mentions of this approximation to $C_{L\alpha}$. It may be recalled that expression for

slope of lift curve of the airplane is obtained in subsection 2.7.1.

Reference 1.8b also expresses

$$C_{m\alpha} = \left(\frac{dC_m}{dC_L}\right)C_{L\alpha}$$

where,

$C_{L\alpha}$ is the slope of the lift curve of the airplane and

$$\frac{dC_m}{dC_L} = \frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}$$

where, x_{ac} is the location of neutral point. There by treating neutral point as the aerodynamic centre of the airplane.

Chapter 2

Lecture 9

Longitudinal stick–fixed static stability and control – 6

Topics

Example 2.4

Example 2.4

Reference 2.4 describes the stability and control data for ten airplanes. This includes a general aviation airplane called "Navion". It seems an appropriate case to illustrate the static stability, dynamic stability and response of an airplane without the complications of compressibility effects. This airplane is dealt with in this chapter and also in chapters 8,9 and 10. The three-view drawing of the airplane is shown in Fig.2.29. The geometrical and aerodynamic data and the flight condition are given below. Some additional data given in Ref.1.1, chapter 2 are also included therein. Remaining data are deduced by measuring dimensions from the three-view drawing. Though references 1.1 and 2.4 use FPS units, data are converted to SI units for the sake of uniformity.

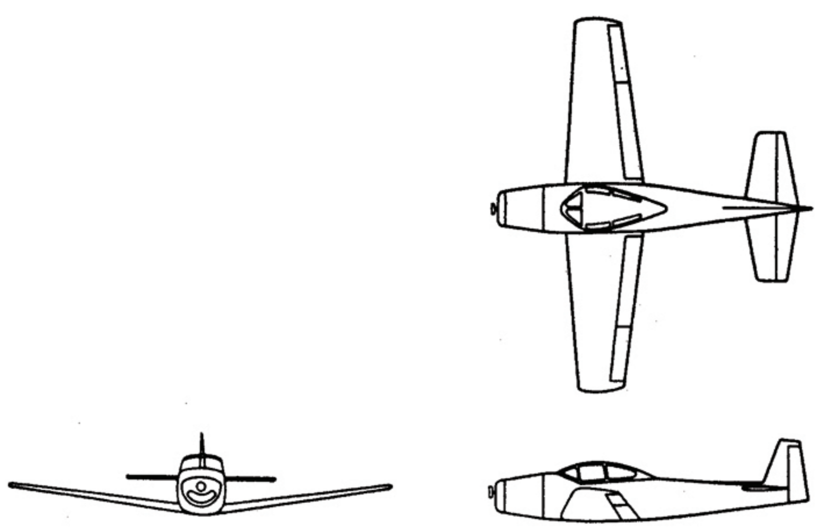


Fig.2.29 Three views of a general aviation airplane

(Adapted from Ref.2.4, section 10)

Wing:

Area (S) = 17.09 m^2, Span (b) = 10.18 m,

Root chord (c_r) = 2.16 m, Tip chord (c_t) = 1.21 m,

Taper ratio (λ) = 0.56, Aspect ratio (A_w) = 6.06,

Mean aerodynamic chord ($\bar{c}$) = 1.737 m,

$i_w = 1^0$ (Ref.1.1, chapter 2):

Characteristics of airfoil used on wing (deduced from Ref.1.1, chapter 2):

$C_{mac} = -0.116$, $C_{l\alpha w} = 0.097\ deg^{-1} = 5.56\ rad^{-1}$, $\alpha_{olw} = -6^0$,

a.c. location $= 0.25\bar{c}$.

Fuselage:

Length (l_f) = 8.23 m,

Width of fuselage at maximum cross section = 1.4 m,

Height of fuselage at maximum cross section = 1.6 m,

The widths at different locations along the length of fuselage are shown in Tables E 2.4.1 & E 2.4.2.

Horizontal tail:

Area (S_t) = 4.73 m^2, Span (b_t) = 4.01 m,

Root chord (c_{rt}) = 1.54 m, Tip chord (c_{rt}) = 0.82 m.

Aspect ratio of tail (A_t)= 3.4,

Distance between quarter chord of the mean aerodynamic chords of wing and tail = 4.63 m, Distance l_h as shown in Fig.2.22 is 3.17 m.

Characteristics of the airfoil used on tail (deduced from Ref.1.1, chapter 2):

$C_{mac} = 0$, $C_{l\alpha t} = 0.1\ deg^{-1} = 5.73\ rad^{-1}$, $i_t = -1^0$.

Flight condition:

Weight = 12232.6 N.

Altitude : sea level, ρ = 1.225 kg/m^3, speed of sound = 340.29 m/s.

flight velocity = 53.64 m/s ; Mach no. (M) = 0.158.

$$\text{Lift coefficient} = C_L = \frac{W}{\frac{1}{2}\rho V^2 S} = \frac{12232.6}{0.5\times1.225\times53.64^2\times17.09} = 0.406;$$

Ref.2.4 gives :

$C_L = 0.41$, c.g. location: $0.295\bar{c}$.

$(C_{L\alpha})_{airplane} = 4.44$, $(C_{m\alpha})_{airplane} = -0.683$.

Obtain : (i) Contributions of wing, horizontal tail, fuselage and power plant to the moment about c.g. (ii) $C_{m\alpha}$ of the airplane (iii) location of the neutral point and (iv) static margin.

Solution:

i) Slopes of lift curve for wing, tail and airplane

From Ref.1.8b, the slope of lift curve for unswept wing at low subsonic Mach number is given by :

$$C_{L\alpha} = \frac{2\pi A}{2+\sqrt{\frac{A^2}{K^2}+4}}; K = \frac{C_{l\alpha}}{2\pi}$$

where, $C_{l\alpha}$ is the lift curve slope of the airfoil.

For wing:

$$C_{L\alpha w} = \frac{2\pi\times 6.06}{2+\sqrt{\frac{6.06^2}{(5.56/6.28)^2}+4}} = 4.17\,\text{rad}^{-1}$$

For horizontal tail:

$$C_{L\alpha t} = \frac{2\pi\times 3.4}{2+\sqrt{\frac{3.4^2}{(5.73/6.28)^2}+4}} = 3.43\,\text{rad}^{-1}$$

The slope of lift curve of the airplane ($C_{L\alpha}$) is obtained using Eq.(2.60b):

$$C_{L\alpha} = C_{L\alpha w} + \eta\frac{S_t}{S}\,C_{L\alpha t}\,(1-\frac{d\varepsilon}{d\alpha})$$

$\eta = 0.9$ is assumed.

$d\varepsilon/d\alpha$ is estimated by the approximate method i.e.

$$\frac{d\varepsilon}{d\alpha} = \frac{2C_{L\alpha w}}{\pi A_w} = \frac{2\times 4.17}{3.14\times 6.06} = 0.438$$

Hence,

$$C_{L\alpha} = 4.17+0.9\times\frac{4.73}{17.09}\times 3.43\,(1-0.438) = 4.65\,\text{rad}^{-1}$$

This estimated value of $C_{L\alpha}$ is only 4.7%higher than the actual values of 4.44 rad^{-1} given in Ref.2.4. Thus, the values of $C_{L\alpha w}$, $C_{L\alpha t}$, $d\varepsilon/d\alpha$ and η are considered to be reasonably accurate (see also Appendix C).

II) Wing contribution:

Following the simplified approach, the wing contributions to C_{m0} and $C_{m\alpha}$ are obtained from Eqs.(2.19) and (2.20):

$$C_{m0w} = C_{macw} + C_{L0w}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right)$$

$$C_{m\alpha w} = C_{L\alpha w}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right)$$

$$C_{Low} = C_{L\alpha w}(i_w - \alpha_{0Lw}) = 4.17\,\frac{\{1-(-6)\}}{57.3} = 0.51$$

$$C_{mow} = C_{macw} + C_{Low}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) = -\,0.116 + 0.51(0.295 - 0.25) = -0.093$$

$$C_{m\alpha w} = C_{L\alpha w}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) = 4.17\,(0.295-0.25) = 0.1887\ \ \text{rad}^{-1}$$

Remark:

For this particular airplane and for the given configuration, the wing contribution to $C_{m\alpha}$ is positive or destabilizing (Note: c.g. is behind a.c.).

III) Horizontal tail contribution:

The tail contributions to C_{m0} and $C_{m\alpha}$ are obtained from the following equations:

$$C_{m0t} = \eta\ V_H\ C_{L\alpha t}\ (i_t - \varepsilon_0)$$

$$(C_{m\alpha t})_{\text{stick-fixed}} = -\eta V_H C_{L\alpha t}\ (1 - \frac{d\varepsilon}{d\alpha})$$

The tail volume ratio is given by:

$$V_H = \frac{S_t}{S}\,\frac{l_t}{\bar{c}} = \frac{4.73}{17.09} \times \frac{4.63}{1.737} = 0.738$$

As estimated earlier : $d\varepsilon/d\alpha = 0.438$

$$\varepsilon_0 = \frac{d\varepsilon}{d\alpha}(i_w - \alpha_{0lw}) = 0.438\{1-(-6)\} = 3.07^0$$

$$C_{m\alpha t} = -V_H \eta C_{L\alpha t}(1-\frac{d\varepsilon}{d\alpha}) = 0.738\times0.9\times3.43\times(1-0.438) = -1.28\ \ \text{rad}^{-1}$$

$$C_{m0t} = -V_H \eta C_{Lt}(i_t - \varepsilon_0) = -0.738\times0.9\times3.43(\frac{-1-3.07}{57.3}) = 0.162$$

IV) Fuselage contribution:

The contributions of fuselage to C_{m0} and $C_{m\alpha}$ are obtained using the method explained in section 2.5.3 and 2.5.4. To obtain C_{mof} we divide the fuselage into nine equal divisions as shown in Fig. 2.30.

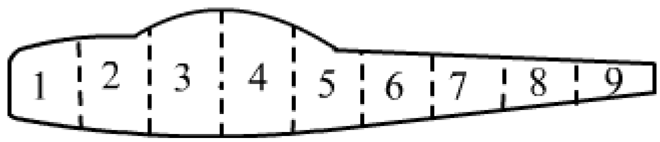


Fig.2.30 Subdivisions of fuselage for calculating C_{mof}

Table E 2.4.1 presents Δx and w_f at various stations along the fuselage. The quantity α_{0Lf} is $i_w + \alpha_{0Lw}$ which equals 1-6 = -5^0. As the fuselage has no camber i_f is taken as zero. Hence, $\alpha_{0Lf} + i_f$ equals -5^0 .The quantity $w_f^2(\alpha_{oLf} + i_f)\Delta x$ is given in the last column of the table E 2.4.1. The sum $\sum w_f^2(\alpha_{0Lf} + i_f)\Delta x$ is - 47.155.

Station	Δx (m)	w_f (m)	$\alpha_{0Lf} + i_f$	$w_f^2(\alpha_{0Lf} + i_f)\Delta x$
1	0.914634	1.097561	-5	-5.497689383
2	0.914634	1.402439	-5	-8.983337807
3	0.914634	1.402439	-5	-8.983337807
4	0.914634	1.402439	-5	-8.983337807
5	0.914634	1.25	-5	-7.141328478
6	0.914634	0.945122	-5	-4.08075913
7	0.914634	0.70122	-5	-2.2387498
8	0.914634	0.457317	-5	-0.963512572
9	0.914634	0.243902	-5	-0.283386051
i_f = 0 at every station				Sum= -47.15543884

Table E 2.4.1 Estimation of C_{m0f}

To obtain the term (k_2-k_1) from Fig.2.19, requires the fineness ratio of the fuselage which is obtained below.

The area of the maximum fuselage cross section (A_{fmax}) is :

A_{fmax} = 1.4 x 1.6 = 2.24 m^2

Hence, equivalent diameter (d_e) is:

$$d_e = \sqrt{A_{fmax}/(\pi/4)} = \sqrt{2.24/(\pi/4)} = 1.69\,m$$

Consequently, fineness ratio = l_f/d_e = 8.23/1.69 = 4.87.   From Fig.2.19, (k_2-k_1) corresponding to fineness ratio of 4.87 is 0.82. Substituting various values , C_{m0f} is given as:

$$C_{m0f} = \frac{k_2 - k_1}{36.5 S\bar{c}} \sum_{x=0}^{l_f} w_f^2 (\alpha_{0Lf} + i_f)\Delta x = \frac{0.82 \times (-47.155)}{36.5 \times 17.07 \times 1.737} = -0.0357$$

To obtain $C_{m\alpha f}$ the fuselage is subdivided as shown in Fig. 2.31. The portion of the fuselage ahead of the root chord is divided into four equidistant portions each of length 0.4573 m. These subdivisions are denoted as 1, 2, 3 and 4. The portion of fuselage aft of the root chord is divided into five equidistant sections each of length 0.8841m and denoted as 5,6,7,8 and 9. The root chord (Fig.2.31) has length c = 1.98 m. Thus, the total fuselage length of 8.23 m is thus divided as: (0.4573 x 4 +1.98 + 0.8841x 5). The length l_h as shown in Fig.2.22 is the distance of the aerodynamic centre of horizontal tail behind the root chord of the wing. It is 3.17m. The calculations of the quantities needed to obtain $C_{m\alpha f}$ are shown in Table E2.4.2. The second column shows Δx which is the length of each subdivision of the fuselage. The third column gives the width of the fuselage in the middle of the subsection (see Fig.2.22). The fourth column gives the distance x for the section 4 as defined in Fig.2.22. For rows 3, 2 and 1 of this column the distance is x_i is as defined in Fig.2.22. For rows 5 to 9 of this column the distance x_i is as shown in Fig.2.22. The fifth column shows $x/\bar{c}$ for the fourth row and $x_i/\bar{c}$ for other rows. The sixth column is dε/dα – the upwash and downwash at the subdivision. For row four the upwash value is based on curve 'b' of Fig.2.23. For rows 3, 2 and 1 the upwash value is based on curve 'a' of Fig.2.23.

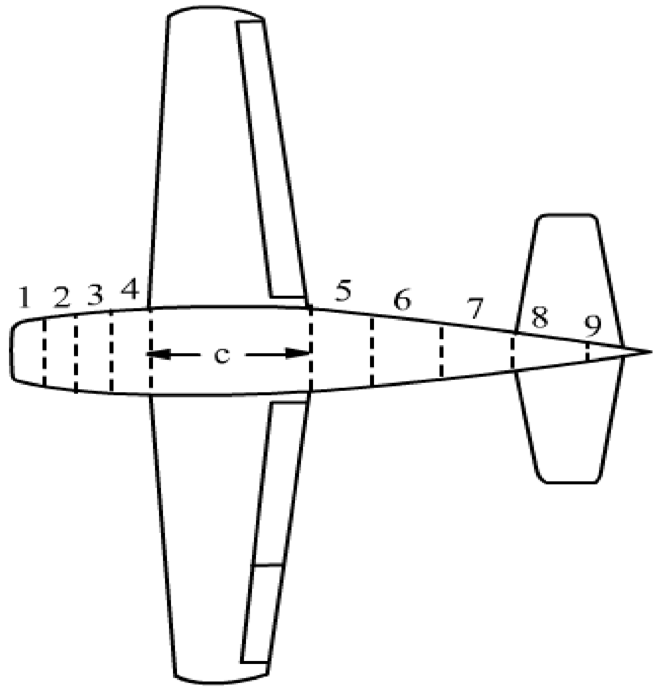


Fig.2.31 Subdivisions of fuselage for estimating $C_{m\alpha f}$

Station	Δx (m)	w_f (m)	x_i or x	(x_i or x)/c	dε/dα*	w_f^2(dε/dα)Δx
1	0.4573	0.914634	1.60061	0.808	1.20	0.4591
2	0.4573	1.036585	1.14329	0.577	1.34	0.6584
3	0.4573	1.158537	0.68598	0.346	1.56	0.9575
4	0.4573	1.280488	0.45731	0.231	3.20	2.4023
5	0.8841	1.158537	0.44207	0.223	0.078	0.0921
6	0.8841	0.945122	1.32622	0.667	0.2341	0.1843
7	0.8841	0.70122	2.21036	1.114	0.3910	0.1715
8	0.8841	0.457317	3.09451	1.561	0.548	0.1011
9	0.8841	0.243902	3.97865	2.008	0.7048	0.03632
c =1.98m		l_h =3.17m				Sum = 5.061

*For $C_{L\alpha W}$ = 0.0785/deg^{-1}. See Remarks (ii) at the end of section 2.5.4

Table E2.4.2 Estimation of $C_{m\alpha f}$

The rows 5 to 9 of this column show the downwash for the corresponding subdivisions. As given in Fig.2.22 and by Eq.(2.58) , dε/dα at the subdivisions behind the root chord is given by:

$$\frac{d\varepsilon}{d\alpha} = \frac{x_i}{l_h}[1 - (\frac{d\varepsilon}{d\alpha})_{tail}]$$

We note that dε/dα at tail is 0.438 for this airplane. Using values of x_i and l_h the values of downwash are tabulated in column 6. The last column shows values of $w_f^2(d\varepsilon/d\alpha)\Delta x$. The sum , $\Sigma w_f^2(d\varepsilon/d\alpha)\Delta x$ is 5.061.

Since, $C_{L\alpha W}$ = 4.17/rad = 0.0728/degree, the actual value of the sum is (see Remark (ii) at the end of section 2.5.4):

5.061 x (0.0728/0.0785) = 4.694.

Finally,

$$C_{m\alpha f} = \frac{k_2 - k_1}{36.5S\bar{c}} \sum_{x=0}^{l_f} w_f^2 \frac{d\varepsilon}{d\alpha}\Delta x = \frac{0.82 \times 4.694 \times 57.3}{36.5 \times 16.56 \times 1.72} = 0.212 \text{ rad}^{-1}$$

V) Contribution of power plant:

It is difficult to estimate this contribution accurately. As mentioned in Remark (ii) in section 2.6.2, this contribution is taken as 0.04 $C_{L\alpha}$ = 0.04 x 4.65 = 0.186.

VI) C_{m0} and $C_{m\alpha}$

The contributions to C_{m0} and $C_{m\alpha}$ from the wing, the horizontal tail, the fuselage and the power plant are shown in Table E 2.4.3. The values of C_{m0} and $C_{m\alpha}$ for the entire airplane are the sums of the values for the components. These are also shown in Table E 2.4.3.

Item	C_{m0}	$C_{m\alpha}$
Wing	-0.093	0.1877
Fuselage	-0.0357	0.212
Power	-	0.186
H.tail	0.162	-1.28
Airplane	0.0333	-0.694

Table E 2.4.3 C_{m0} and $C_{m\alpha}$ due to components and for the entire airplane

Figure 2.27 shows the contributions graphically. C_{mcg} and $C_{m\alpha}$ for the airplane are:

$$C_{mcg} = -0.093 + 0.1877\alpha - 0.0357 + 0.212\alpha + 0.186\alpha + 0.1620 - 1.28\alpha$$
$$= 0.0333 - 0.694\,\alpha$$

Hence, $(C_{m\alpha})_{\text{stick-fixed}} = -0.694$

It is very interesting to note that the value of $C_{m\alpha}$ given in Ref.2.4 is -0.683. Thus the estimates of the contributions of various components to static stability can be considered to be reasonably accurate.

VII) Neutral point location:

The neutral point is given by:

$$\frac{x_{NP}}{\bar{c}} = \frac{x_{ac}}{\bar{c}} - \frac{1}{C_{L\alpha w}}\{(C_{m\alpha})_{f,n,p} - V_H\ \eta\ C_{L\alpha t}(1-\frac{d\varepsilon}{d\alpha})\}$$

Substituting various values, $\frac{x_{NP}}{\bar{c}}$ is given as:

$$\frac{x_{NP}}{\bar{c}} = 0.25 - \frac{1}{4.17}\{0.212 + 0.186 - 1.28\}$$
$$= 0.25 + 0.2115 = 0.4615$$

VIII) The static margin:

The static margin when c.g. is at $0.295\bar{c}$ is :

$$\frac{x_{NP}}{\bar{c}} - \frac{x_{cg}}{\bar{c}} = 0.4615 - 0.295 = 0.1665.$$

Hence, $(dC_m/ dC_L) = -(\text{static margin}) = -0.1665$

$C_{m\alpha} = C_{L\alpha w}\,(dC_m / dC_L) = 4.17 \times (-0.1665) = -0.694$ as it should be.

Chapter 2

Longitudinal stick–fixed static stability and control

Lecture 10

Topics

Example 2.5

Example 2.6

2.12 Longitudinal control

2.12.1 Elevator power

2.12.2 Control effectiveness parameter (τ)

2.12.3 Elevator angle for trim

2.12.4 Advantages and disadvantages of canard configuration

2.12.5 Limitations on forward movement of c.g. in free flight

2.12.6 Limitations on forward movement of c.g. in proximity of ground

Example 2.5

A sailplane has the following characteristics. $C_D = 0.02 + 0.025\ C_L^2$, $C_{L\alpha w} = 0.093$, $\alpha_{0Lw} = -4$, $i_w = 0$, a.c. location $= 0.24\bar{c}$, $S_t = S/7$, $l_t = 4\bar{c}$, $\varepsilon = 0.4\alpha$, $C_{L\alpha t} = 0.05$ and $\eta = 0.9$. All the angles are in degrees. Neglect the contribution of fuselage. Find the c.g. location for which the equilibrium is reached with zero lift on the tail at the lift coefficient corresponding to the best guiding angle. Calculate the tail setting. Is the sailplane stable?

Solution:

The airplane prescribed in this exercise is a sailplane. A sailplane is a high performance glider. There is no power plant in a glider. Further, the contribution of fuselage is prescribed as negligible. Hence, the terms $(C_{mcg})_{f,n,p}$ and $(C_{m\alpha})_{f,n,p}$ are zero in the present case.

The given data is as follows.

$$C_D = C_{D0} + KC_L^2 = 0.02 + 0.025\ C_L^2$$

Wing: $C_{Lw} = 0.093\ (\alpha_w + 4)$, $C_{L\alpha w} = 0.093\ \text{deg}^{-1} = 5.329\text{rad}^{-1}$, $C_{mac} = -0.08$, $i_w = 0$, a.c. at $0.24\bar{c}$.

Tail: $S_t = S / 7$, $\varepsilon = 0.4\ \alpha$ or $d\varepsilon/d\alpha = 0.4$, $\eta = 0.9$, $C_{L\alpha t} = 0.05\ \text{deg}^{-1} = 2.865\ \text{rad}^{-1}$.

For the best gliding angle (C_D/C_L) should be minimum.

This happens when $C_L = C_{Lmd}$ and $C_{Lmd} = \sqrt{C_{D0}/K}$

In the present case $C_{Lmd} = \sqrt{0.02/0.025} = 0.895$

When the airplane is flying at $C_L = C_{Lmd}$, the lift on tail is prescribe to be zero or $\alpha_t = 0$.

Now, $\alpha_t = \alpha_w - i_w - \varepsilon + i_t$,

At $C_L = C_{Lmd}$, $\alpha_w = (0.895/ 0.093) - 4 = 5.62^0$, $\varepsilon = 0.4\ \alpha = 0.4\ (\alpha_w - i_w)$

At $C_L = C_{Lmd}$, $\varepsilon = 0.4\ (5.62 - 0) = 2.25^0$

Since, α_t is zero at $C_L = C_{Lmd,}$ gives the following result.

$0 = 5.62 - 2.25 + i_t$

or $i_t = - 3.37^0$

To examine static stability, the quantity $(C_{m\alpha})_{stick\text{-}fix}$ is calculated. It is noted that:

$C_{mcg} = C_{m0} + C_{m\alpha}\ \alpha$

$$C_{mcg} = C_{mac} + C_{Lw}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) + (C_{mcg})_{f,n,p} + C_{mcgt}$$

$$(C_{m\alpha})_{stick\text{-}fix} = C_{L\alpha w}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) + (C_{m\alpha})_{f,n,p} - \eta V_H C_{L\alpha t}\left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

To evaluate the expression for $C_{m\alpha}$ the quantity $\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right)$ is needed. This can be obtained using the following steps.

When the airplane is flying at $C_L = C_{Lmd}$, the contributions of tail, fuselage, nacelle and power are zero. Hence, the expression for C_{mcg} reduces to

$$C_{mcg} = C_{Lw}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) + C_{mac}$$

For equilibrium C_{mcg} must be zero at C_{Lmd} i.e. :

$$0 = 0.895\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) - 0.08$$

$$\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}} = \frac{0.08}{0.895} = 0.089$$

$$\text{or } \frac{x_{cg}}{\bar{c}} = 24 + 0.089 = 0.329$$

$$\text{Further, } V_H = \frac{S_t}{S}\frac{l_t}{\bar{c}} = \frac{4}{7}$$

$$\text{Finally, } C_{m\alpha} = 5.329(0.329 - 0.24) - 0.9 \times \frac{4}{7} \times 2.865 \times (1 - 0.4)$$

$$= 0.47428 - 0.88406 = -0.4098 \text{ rad}^{-1}$$

$C_{m\alpha}$ is negative and hence the sailplane is stable.

Example 2.6

The contribution of wing fuselage combination to the moment about the c.g. of an airplane is given below.

C_L	0.28	0.488	0.696	0.9
$(C_{mcg})_{w,f}$	- 0.0216	- 0.006	0.0064	0.0156

(i) If the wing loading is 850 N/m^2, find the flight velocity at sea level when the airplane is in trim with zero lift on the tail. (ii) Investigate the stability of the airplane with the following additional data: $C_{L\alpha w}$ = 0.08 deg^{-1}, $C_{L\alpha t}$ = 0.072 deg^{-1}, $d\varepsilon/d\alpha = 0.45$, $l_t = 2.9\bar{c}$, S_t = S/7, η=1.0. Assume the contributions of power to C_{mcg} and $C_{m\alpha}$ to be negligible.

Solution:

i) To answer the first part, the value of C_L at which the airplane is in trim with zero lift on tail needs to be obtained. In this case:

$$C_{mcg} = (C_{mcg})_{w,f} = 0$$

The prescribed variation of $(C_{mcg})_{w,f}$ with C_L is slightly non-linear. Hence, the given data are plotted and the value of C_L at which $(C_{mcg})_{w,f}$ is zero is obtained from the plot. The plot is shown in Fig. E2.6. When $(C_m)_{w,f}$ is zero, C_L equals 0.585 .

In level flight: $L = W = \frac{1}{2}\rho V^2 S C_L$

or $V = \sqrt{2W/SC_L}$

Substituting various values, the desired velocity is:

$$V = \sqrt{\frac{2\times 850}{1.225\times 0.585}} = 48.7\,\text{ms}^{-1}$$

ii) To examine the static stability $C_{m\alpha}$ needs to be calculated.

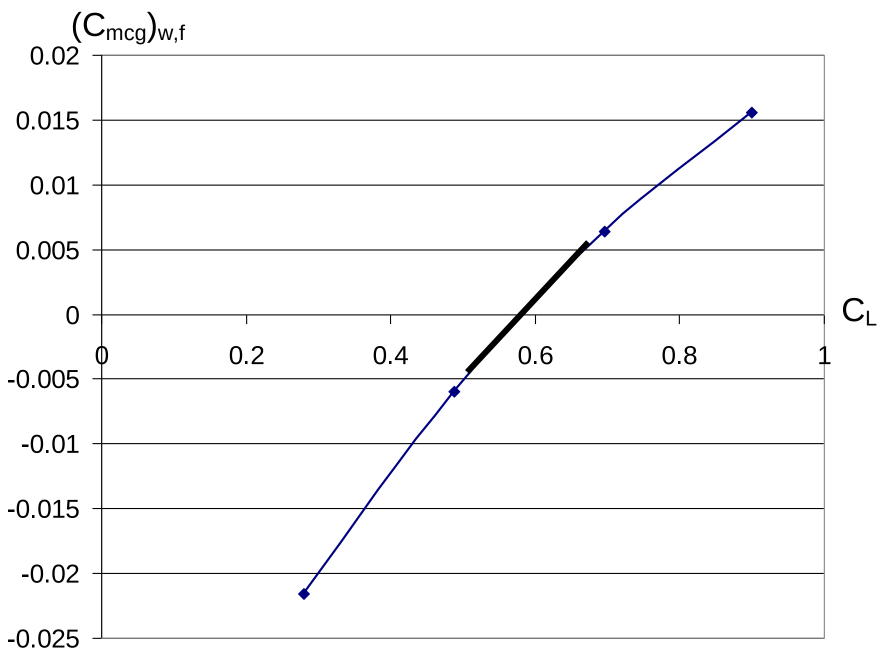


Fig.E2.6 $(C_{mcg})_{w,f}$ vs C_L

$$C_{m\alpha} = C_{L\alpha w}\left(\frac{dC_m}{dC_L}\right)_{w,f} - V_H\,\eta\,C_{L\alpha t}\left(1-\frac{d\varepsilon}{d\alpha}\right)$$

$$C_{L\alpha w} = 0.08, C_{L\alpha t} = 0.072,\ V_H = \frac{1}{7}\times 2.9 = 0.414$$

$$\frac{d\varepsilon}{d\alpha} = 0.45$$

From graph in Fig.E2.6 at $C_L = 0.585$ we obtain, the slope of the curve as:

$(dC_m/dC_L)_{w,f} = 0.0615$

Hence, $C_{m\alpha} = 0.08 \times 57.3 \times 0.0615 - 0.414 \times 1 \times 0.072 \times 57.3 \times (1 - 0.45)$

$= 0.2819 - 0.9394 = - 0.6575 \ \text{rad}^{-1}$

Since, $C_{m\alpha}$ is negative the airplane is stable.

2.12 Longitudinal control

An airplane is said to be trimmed at a given flight speed and altitude, when the moments are made zero by suitable deflection of control surfaces. For the longitudinal motion, the trim or $C_{mcg} = 0$ is achieved by suitable deflection of elevator. The convention regarding the elevator deflection is that a downward deflection of elevator is taken as positive (Fig.2.16b). For the conventional tail configuration, this deflection increases lift on tail and produces a negative moment about c.g..

Let ΔC_L and ΔC_{mcg} be the incremental lift and pitching moment due to the elevator deflection i.e.

$$\Delta C_L = \Delta C_{Lt} = C_{L\delta e}\, \delta_e \ ; C_{L\delta e} = \frac{\partial C_L}{\partial \delta_e} \tag{2.72}$$

$$\Delta C_{mcg} = \Delta C_{mcgt} = C_{m\delta e} \times \delta_e \ ; C_{m\delta e} = \frac{\partial C_{mcg}}{\partial \delta_e} \tag{2.73}$$

Hence, when the elevator is deflected, the lift coefficient (C_L) and moment coefficient about c.g. (C_{mcg}) for the airplane are :

$$C_L = C_{L\alpha}(\alpha - \alpha_{0L}) + C_{L\delta e}\, \delta_e \tag{2.74}$$

$$C_{mcg} = C_{m0} + C_{m\alpha}\, \alpha + C_{m\delta e}\, \delta_e \tag{2.75}$$

Where C_L, $C_{L\alpha}$ and α_{0L} refer respectively to the lift coefficient, slope of the lift curve and zero lift angle of the airplane.

Note: In this section $C_{m\alpha}$ will mean $(C_{m\alpha})_{\text{stick-fixed}}$.

2.12.1 Elevator power ($C_{m\delta e}$)

The quantity $C_{m\delta e}$ is called elevator power. An expression for it has been hinted in Eq.(2.64). It can be derived as follows.

Let, $\Delta L_{\delta e}$ be the change in the airplane lift due to elevator deflection which is also the change in the lift of the horizontal tail i.e.

$$\Delta L_{\delta e} = (\Delta L_t)_{\delta e} = \frac{1}{2}\rho\, V_t^2\, S_t(\Delta C_{Lt})_{\delta e}$$

$$\Delta C_{L\delta e} = \frac{\Delta L_{\delta e}}{\frac{1}{2}\rho V^2 S} = \eta\frac{S_t}{S}(\Delta C_{Lt})_{\delta e} = \eta\frac{S_t}{S}\frac{\partial C_{Lt}}{\partial \delta_e}\delta_e \qquad (2.76)$$

$$\text{Hence, } \frac{\partial C_L}{\partial \delta_e} = C_{L\delta e} = \eta\frac{S_t}{S}\frac{\partial C_{Lt}}{\partial \delta_e} \qquad (2.77)$$

$$\Delta M_{\delta e} = \Delta L_{\delta e} l_t = \frac{1}{2}\rho V_t^2 S_t(\Delta C_{Lt})_{\delta e} l_t$$

$$\Delta C_{m\delta e} = \frac{\Delta M_{\delta e}}{\frac{1}{2}\rho V^2 S\bar{c}} = \frac{\frac{1}{2}\rho V_t^2}{\frac{1}{2}\rho V^2}\frac{S_t}{S}\frac{l_t}{\bar{c}}(\Delta C_{Lt})_{\delta e}$$

$$\text{Or } \Delta C_{m\delta e} = -V_H\, \eta\, (\Delta C_{Lt})_{\delta e} = -V_H\, \eta\frac{\partial C_{Lt}}{\partial \delta_e}\delta_e$$

$$\text{Hence, } \frac{\partial C_m}{\partial \delta_e} = C_{m\delta e} = -V_H\,\eta\frac{\partial C_{L_t}}{\partial \delta_e} = -V_H\,\eta\,\tau\, C_{L\alpha t};\ \tau = C_{L\delta e}/C_{L\alpha t} \qquad (2.78)$$

2.12.2 Control effectiveness parameter (τ)

The quantity 'τ' is called elevator effectiveness parameter. The value of τ depends on the geometrical parameters of the tail and the elevator. However, it mainly depends on (S_e / S_t) where S_e is the area of the elevator. References 1.12 and 2.2 give a detailed procedure for estimating it. However, Fig.2.32 can be used for an initial estimate.

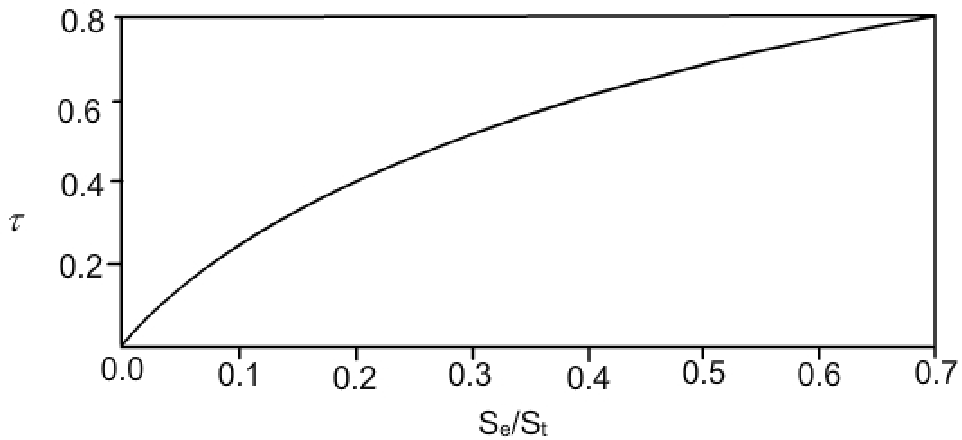


Fig.2.32 Control effectiveness parameter

(Reproduced from Refs.1.1, chapter 2 with permission from McGraw-Hill book company)

2.12.3 Elevator angle for trim

The following steps are followed to get the elevator angle for trim (δ_{etrim}).

From Eq. (2.75)

$$C_{mcg} = C_{m0} + C_{m\alpha}\,\alpha + C_{m\delta e}\,\delta_e$$

For trim $C_{mcg} = 0$.

Hence,

$$0 = C_{m0} + C_{m\alpha}\,\alpha_{trim} + C_{m\delta e}\,\delta_{trim}$$

Or $$\delta_{trim} = \frac{-1}{C_{m\delta e}}[C_{m0} + C_{m\alpha}\,\alpha_{trim}] \qquad (2.79)$$

From Eq.(2.74)

$$C_{Ltrim} = C_{L\alpha}(\alpha_{trim} - \alpha_{0L}) + C_{L\delta e}\,\delta_{trim} \qquad (2.80)$$

Or $$\alpha_{trim} = \frac{1}{C_{L\alpha}}\{C_{Ltrim} - C_{L\delta e}\,\delta_{trim} + C_{L\alpha}\,\alpha_{0L}\}$$

Hence,

$$\delta_{trim} = \frac{-1}{C_{m\delta e}}[C_{m0} + \frac{C_{m\alpha}}{C_{L\alpha}}\{C_{Ltrim} - C_{L\delta e}\,\delta_{trim} + C_{L\alpha}\,\alpha_{0L}\}] \qquad (2.81)$$

Or $$C_{m\delta e}\,\delta_{trim} = -\frac{[C_{m0}\,C_{L\alpha} + C_{m\alpha}(C_{Ltrim} + C_{L\alpha}\alpha_{0L}) - C_{m\alpha}\,C_{L\delta e}\,\delta_{trim}]}{C_{L\alpha}}$$

Simplifying, $$\delta_{trim} = -\frac{[C_{L\alpha}\,(C_{m0} + C_{m\alpha}\,\alpha_{0L}) + C_{m\alpha}\,C_{Ltrim}]}{[C_{m\delta e}\,C_{L\alpha} - C_{m\alpha}\,C_{L\delta e}]} \tag{2.82}$$

Differentiating with C_L ,

$$\frac{d\delta_{trim}}{dC_{Ltrim}} = -\frac{C_{m\alpha}}{[C_{m\delta e}\,C_{L\alpha} - C_{m\alpha}\,C_{L\delta e}]} \tag{2.83}$$

Following may be noted.

(i) The quantities $C_{m\delta e}$, $C_{L\alpha}$ and $C_{L\delta e}$ depend on the airplane geometry. Further, for a given c.g. location, $C_{m\alpha}$ is also known and hence the term ($d\delta_{etrim}/dC_{Ltrim}$) in Eq.(2.83) is a constant. Hence, in the simplified analysis being followed here, it is observed that for a given c.g. location δ_{trim} is a linear function of C_L. Further the term $C_{m\alpha}\,C_{L\delta e}$ is much smaller than $C_{m\delta e}\,C_{L\alpha}$. Consequently, δ_{trim} as a function of C_L can be written as:

$$\delta_{trim} = \delta_{eoCL} - \frac{1}{C_{m\delta e}}\left(\frac{dC_m}{dC_L}\right)_{stick\text{-}fixed} C_L \tag{2.84}$$

where,

$$\delta_{e0CL} = -\frac{C_{L\alpha}\,(C_{m0} + C_{m\alpha}\,\alpha_{0L})}{[C_{m\delta e}\,C_{L\alpha} - C_{m\alpha}\,C_{L\delta e}]}$$

Similarly, using Eq.(2.79), δ_{trim} as a function of α_{trim} can be expressed as:

$$\delta_{trim} = -\frac{C_{m0}}{C_{m\delta e}} - \frac{C_{m\alpha}}{C_{m\delta e}}\alpha_{trim} \tag{2.85}$$

Typical curves for the variations of δ_{trim} with C_L are shown in Fig.2.33 for different locations of c.g.. From Eq.(2.85) it is seen that at $C_L = 0$ the value of δ_{trim} is positive as C_{m0} is positive and $C_{m\delta e}$ is negative. For a stable airplane $(dC_m/dC_L)_{stick\text{-}fixed}$ is negative and hence the slope of δ_{trim} vrs C_L curve is negative.

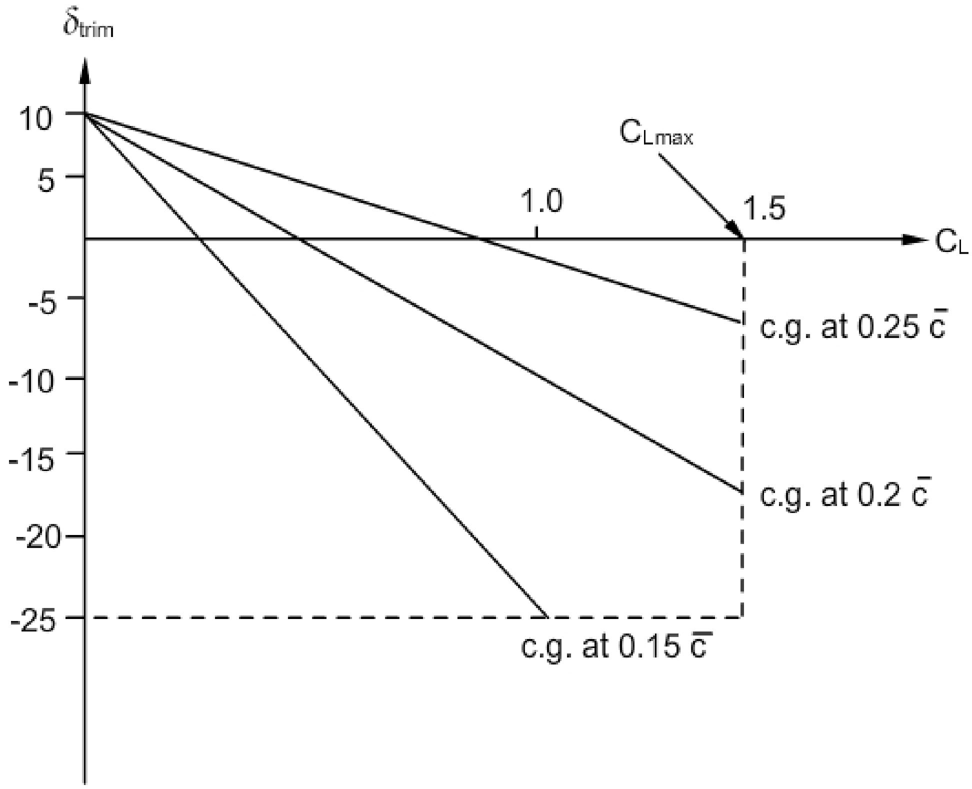


Fig.2.33 Elevator angle for trim

(ii) In light of the above analysis, consider a case when the airplane is trimmed at a chosen C_L by setting the elevator at corresponding $\delta_{trim.}$ Now, if the pilot wishes to fly at a lower speed which implies higher C_L, he would need to apply more negative elevator deflection or the incremental lift on the tail (ΔL_t) would be negative. This is what is implied when in section 2.4.1 it is mentioned that "… for achieving equilibrium with conventional tail configuration, the lift on the tail is generally in the downward direction". An alternate explanation is as follows.

When the pilot wishes to increase the angle of attack by $\Delta\alpha$, a statically stable airplane produces a moment $-\Delta M_{cg}$.To counterbalance this moment, the elevator must produce $+\Delta M_{cg}$. This requires $-\Delta L_t$ and inturn $-\Delta\delta_e$.

(iii) Military airplanes which are highly maneuverable, sometimes have the following features.

(a) An all movable tail in which the entire horizontal tail is rotated to achieve higher ΔM_{cg}. (b) Relaxed static stability wherein $C_{m\alpha}$ may have a small positive value. Such airplanes need automatic control (section 10.3). See section 6.1 of Ref.1.13 for further details.

2.12.4 Advantages and disadvantages of canard configuration

In light of the above discussion the advantages and disadvantages of the canard configuration can now be appreciated.

Advantages:

(a) The flow past canard is relatively free from wing or engine interference.

(b) For an airplane with $C_{m\alpha} < 0$, the lift on the horizontal stabilizer located behind the wing (i.e. conventional configuration) is negative when the angle of attack increases. Thus, for a conventional tail configuration, the wing is required to produce lift which is more than the weight of the airplane. If the surface for control of pitch, is ahead of the wing (canard), the lift on such horizontal control surface is positive and the lift produced by the wing equals the weight of the airplane minus the lift on canard. Thus, the wing size can be smaller in a canard configuration.

Disadvantages:

(a) The contribution of the canard to $C_{m\alpha}$ is positive i.e. destabilizing.

(b) As the wing, in this case, is located relatively aft, the c.g. of the airplane moves aft and consequently the moment arm for the vertical tail is small.

Topics for self study:

1. From Ref.2.3 study the airplanes with canard and obtain rough estimates of (S_t / S) and (l_t / c). Two examples of airplanes where canard is used are SAAB Viggen and X-29A.

2.12.5 Limitations on forward moment of c.g. in free flight

As the c.g. moves forward, the airplane becomes more stable and hence requires larger elevator deflection for trim at a chosen C_L. It is seen that as C_L increases, more negative elevator deflection is required (Fig. 2.33). Further, each airplane has a value of C_{Lmax} which depends on the parameters of the wing. However, equilibrium at C_{Lmax} can be achieved only if the airplane can be trimmed at this lift coefficient. Further, the maximum elevator deflection is limited to approximately about 25^0(negative). Hence, there would be a forward c.g. location at which the maximum negative elevator deflection would be just able to permit trim at C_{Lmax}. This brings about a limitation on the forward movement of c.g. from control consideration. It may be recalled that the rearword movement of c.g. is limited by the stability consideration.

2.12.6 Limitations on forward movement of c.g in proximity of ground

As the airplane comes in to land, the lift coefficient is generally the highest. It is achieved using flaps and this makes C_{macw} more negative. Further, due to the proximity of ground, following changes in $C_{L\alpha w}$ and $(d\varepsilon / d\alpha)$ are observed.

(a) The slope of lift curve of the wing, i.e. $C_{L\alpha w}$ increases slightly. The actual amount of increase in $C_{L\alpha w}$ depends on the ratio of the height of the wing above the ground and its span (see Ref.1.7, chapter 5). There is no significant change in $C_{L\alpha t}$.

(b) The downwash due to wing decreases considerably (Fig.2.12) and consequently the tail contribution to stability ($C_{m\alpha t}$) becomes more negative (Eq.2.50) or the airplane becomes more stable.

The net effect is that the airplane requires more negative elevator deflection. This imposes further restrictions on the forward movement of c.g.. Figure 2.34 shows the restrictions on c.g. travel based on factors discussed so for. Additional restrictions on the movement of c.g. would be pointed out after discussions in chapters 3 and 4.

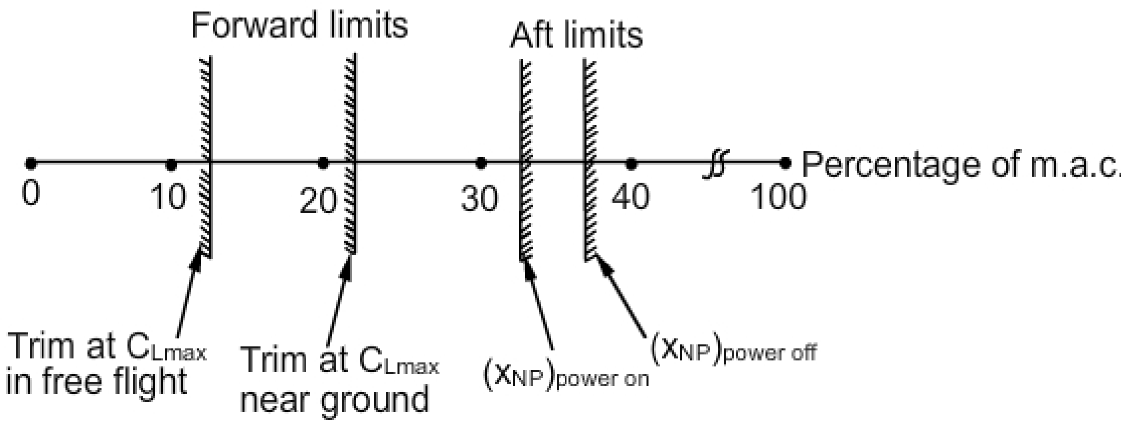


Fig.2.34 Restrictions on c.g. movement from stick fixed stability and control considerations (schematic)